ESTIMATION OF AN ACROSS-INDEX COMMON FACTOR: AN APPLICATION OF A DYNAMIC COMMON FACTOR MODEL TO STOCK INDICES

by

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Contents

1. Introduction / Related Research

Economic theories have been proposed to suggest that asset returns can be distilled into single or multi-factor models. These factors often relate the return to the risk inherent in the asset. The most famous theories include the arbitrage pricing theory (APT) of Ross (1976), and the capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965) and Merton (1973). Extensions to these theories have included consumption based CAPM (CCAPM) models by Breeden (1989) and Lucas (1978).

The APT is a factor model that assumes a set of factors exist that determine an asset's rate of return. The rate of return for a particular asset can be viewed as compensation for the risk inherent in the asset. These factors are systematic (undiversifiable) factors which affect the asset's return. The commonly referred to beta coefficient explains the impact of such factors to the asset's return.

The CAPM model is a simplification of APT. It is a theory of financial equilibrium only and does not attempt to link an asset's return with events in the real side of the economy. CAPM suggests that the appropriate measure of an asset's risk is the beta coefficient which is defined as the covariance of the asset's return with the market return. CAPM is a single factor model that explains asset returns using the market risk premium. The return of an asset can be explained with two variables: the expected risk-free return and the market risk premium. The market risk premium is the expected market return less the expected risk-free return. The beta coefficient explains the asset's relation to systemic or market risk as represented by the market risk premium. The CCAPM is an extension to traditional CAPM which ties the interaction of the real macro-economy and asset markets. CCAPM suggests that an appropriate measure of an asset's risk is the covariance with aggregate consumption.

However, a theory is essentially an idea that should be refutable. In order for an economic theory to be accepted as an explanation of the real world movements of the market, these theories must be empirically tested. Here lies the problem, the seminal papers which provide these theories are often silent on how many or which variables the market participants employ. If APT is to be tested, the risk factor(s) common to all assets must be identified. For CAPM, most studies use a stock market index as a proxy for the market portfolio. In the Mankiw and Shapiro (1987) paper, they compare the theories of CAPM and CCAPM. In order to perform their analysis, they parameterize the models using the Standard and Poor's 500 index to represent the market and the real consumer expenditure on non-durables and services to represent the consumption measure. In the Chen, Roll and Ross (1986) paper which tests whether innovations in macroeconomic variables are risks that are rewarded in the stock market, among the variables the authors used were the equal and value weightings of the NYSE index to represent the market. In the Connor and Korajczyk (1991) paper which investigated the risk and return characteristics of US mutual funds they also used the value-weighted portfolio of the NYSE stocks available from the Center for Research in Security Prices to represent the market portfolio.

However, the theoretical model implied that the market portfolio includes all possible assets. The same problem exists for CCAPM, where a measure for consumption must be defined. Mankiw and Shapiro's (1987) paper notes that possible objections may arise from their choice of proxy variables to represent the market portfolio; this is one example which highlights the importance of selecting factors in both the APT and CAPM framework.

Perhaps the most famous attempt to identify factors to explain stock returns is by Fama and French (1993). In their paper they extended the CAPM model by adding two observable risk factors; the small minus big (SMB) factor and the high minus low (HML) factor. The SMB factor mimics the risk in returns related to size and the HML factor is meant to mimic the risk related to the book-to-equity measure of stocks.

Some economists argue that it is impossible to test asset pricing theories. Richard Roll's (1977) famous paper postulates that the market portfolio cannot be identified as the exact composition of the true market portfolio is unknown. This implies that CAPM is not-testable unless all individual assets are included in the sample. He further argues that most reasonable proxies will be highly correlated with each other and the market portfolio and that this correlation will make it seem that the exact composition is unimportant, whereas it can cause quite different inferences. From the plethora of research, it is clear that identification of factors is important in the explanation of financial assets.

Even though there have been noted issues with beta factor models, finance practitioners continue to use the models for two reasons: risk measurement and portfolio selection. Estrada and Vargas' (2011) paper investigates the risk measurement premises by determining whether high-beta portfolios of countries and industries fall more than low-beta portfolios when exposed to large market declines. They also explore whether beta is a valuable tool for portfolio selection. Using data spanning 47 countries, 57 industries, and four decades they determined that beta is a useful measure of risk in the sense of accounting for exposure to the downside, and in particular, to large and unexpected market declines. They also test an investment strategy that reacts to large unexpected market declines by investing in high-beta portfolios, and to large positive market increases by investing in low-beta portfolios and conclude that their beta-based strategy outperforms a passive investment strategy. Estrada and Vargas conclude that beta is a valuable tool for portfolio selection.

Recall that a beta model framework, such as CAPM, is used to relate the return of an asset to the risk associated to the market portfolio which includes all investable assets. Often, a particular stock market index such as the Dow Jones Industrial Index is used as a proxy for risk associated to the market. The determination of the betas of a common factor for the multiple stock indices and the common composite market index is an increasingly important issue as more investors are able to invest in diversified portfolios across stock exchanges both locally and internationally. Investors are now able to readily invest in assets which span different countries and industries through instruments such as low-cost funds and exchange traded funds. The individual stock index betas of the common factor will allow investors to determine the risk exposure of the composite market index, enabling investors to understand which regions or industries or assets are more risky.

The contribution of this paper is to attempt to determine a single factor for a broad set of market indexes which can in turn be used in simple CAPM framework for stock index returns. Specifically, this paper attempts to identify the existence of a risk factor that is common to multiple stock indices.

Similar work has been performed by Vansteenkiste (2009) where commodity prices were analyzed. Using a dynamic factor approach, she was able to separate common and idiosyncratic developments of commodity prices. She also concluded that over time the common factor has recently become more important in explaining the movements of commodity prices.

With the increasingly globalized world, the transmission of shocks from one stock market can rapidly occur. Such co-movements in different stock indices have important implications for setting economic policy. Furthermore, the determination of a common risk factor is an important consideration for many portfolio diversification techniques.

Following the analysis of Vansteenkiste (2009), this paper will analyze whether idiosyncratic factors or a coinciding of individual stock index shocks have been driving the stock market returns. The coinciding factor is the estimated beta coefficient on a common factor for the stock indices. To accomplish this, we will use a dynamic common factor model to analyze the significance of a common factor to explain the major North and South American stock index returns. While the stock markets and the related indices may have slightly varied operating hours, and different geographic locations, they do have a degree of operating overlap. Therefore, there is a degree of logic that the movements in these stock indices will be affected by a common factor. With the results we can also utilize a rolling-window analysis to determine how the significance of the common factor for each stock index changes over time. In addition, we will compare the betas of individual stocks from their standard stock indices against the betas associated to a common composite index. We will also test the assertion that overlapping operating hours is an important factor by constructing a dynamic common factor model which includes indices from North America, Europe and Asia.

In this paper, we argue that the dynamic common factor framework is helpful and provides additional information compared with using an existing composite index or using a leading eigenvector in principle component analysis. The dynamic common factor model using a Kalman filtering technique estimates an unobservable common factor which can be loosely interpreted as an across-exchange market composite index with suitable scaling.

The various individual exchange composite indices such as those offered by MSCI can be used as a "market" risk factor to explain the returns of particular stock indices. Such composite indices are constructed along various guidelines such as country, size, sector, etc. These indices are created from the inclusion of individual assets and weighting them accordingly. The use of a dynamic factor framework allows us to include the specific regions or indices to construct a custom composite index which allows for a broader and more general risk analysis against comparisons of an individual stock, portfolio or index.

In comparison to using a leading eigenvector principal component analysis (PCA), the dynamic common factor model allows for measurement of the co-movement of time-series data and accounts for the evolution of the common and idiosyncratic components. The PCA approach has two differences. First, the approach relies on the determination of eigenvectors and value of the covariance matrix. The covariance matrix and the co-movement of the data extract the contemporaneous correlations of the data series, ignoring the time-series component. Therefore, the dynamic factor model may dominate if there is even a slight persistence to the indexes. Secondly, using the leading eigenvector in a PCA analysis reduces the dimensionality of the data. Since PCA wills us to express data in terms of the patterns between each data set; only using the leading eigenvector to transform the data will remove the contribution of the other eigenvectors. Should persistence exist, the use of a dynamic factor framework will allow us to explain the co-movements and determine the impact from each individual data set. However, should the expected returns of the common factor exhibit low persistence, the common component could be very close to the principal component leading eigenvector.

The use of a dynamic common factor framework will allow for the construction of broader beta models that will allow individual stocks to be gauged against a full cross-market option. As alluded to in the comparison against an existing composite index, the dynamic factor framework will allow practitioners to estimate a common factor which could be interpreted as a market composite index customized to what they perceive as market risk. The resulting analysis will allow the practitioner to interpret the risks by determining which portion of risk is common to all markets and which portion is specific to a particular market.

The results of this paper show that there exists a significant common factor or market composite index which accounts for a portion of the logarithmic daily returns for each stock index. We also reject the null hypothesis that impact coefficients are jointly zero, which suggests that the common factor accounts for a portion of the returns for all indices. In other words, there is a degree of co-movement of the stock indices attributed to the common factor. Due to the identifying orthogonality restrictions placed on the model, economic interpretation of the coefficients is provided through a variance decomposition. The analysis shows that the common factor's variance accounts for at most 50% (US indices) of the overall variance. A rolling two-year correlation with the common factor is also performed and the results show that over the sample period the correlation has increased. The analysis for individual stock's beta showed that using a standard index (SP500 or NASDAQ) overstates the impact of the standard index as compared to the common composite index.

When the model was applied to include North American, European, and Asian indices, the variance decomposition showed that the percentage of variance accounted for by the common factor is geographically grouped, with a higher percentage for those indices which overlap in operating hours. This suggests that when indices overlap their common factor beta is higher.

The empirical work is primarily conducted with daily data from April 28, 1993 to December 31, 2010, for Standard & Poor 500, NASDAQ, Russell 2000, S&P/TSX, BOVESPA, and IPC indices and expanded with the FTSE, DAX, Hang Seng, and Nikkei 255 indices. Section II describes the econometric theory behind the dynamic factor analysis. Section III describes the data sources, modifications to the data, and tests to determine the time-series data has desirable qualities for analysis. Section IV describes the results of the dynamic common factor analysis. Section V concludes the paper and provides a discussion of possible extensions to the research.

2. The Statistical Model

The dynamic common factor model is applied to the time-series stock index data using Kalman filtering techniques. This methodology is commonly used in the world business cycle literature. Examples of this include papers by Gregory, Head and Raynauld (1997), Stock and Watson (1989), and Stock and Watson (1991). In general, as described in Lütkepohl (2005) the dynamic factor models represent a vector of k endogenous observable variables, y_t, as linear functions of $n < k$ unobserved common factors, f_t, and some idiosyncratic components, u_t. In some models there are no idiosyncratic terms.

The model can be represented as:

$$y_t = Lf_t + u_t,$$

where L is a $(K\ x\ N)$ matrix of factor loadings and the components u_t are assumed to be uncorrelated, that is, the covariance matrix of u_t is diagonal. The unobserved factors, f_t, and the disturbances, u_t, in the equations for the observed variables may follow vector autoregressive structures.

The advantage of the dynamic factor model is the ability to measure the co-movement of the variable with time dependencies, as opposed to just the contemporaneous correlations using a PCA analysis. In addition, the analysis distinguishes between the idiosyncratic component and the common component as the source of the time-series co-movements.

The approach analogous to the CAPM with a single index is the use of capturing the co-movements in the indices are a common risk factor that can be captured by a single latent variable. Since the model is linear in the unobserved variable, the Kalman filter is used to construct the Gaussian likelihood function, which allows us to estimate a composite market index which is a composition of all stock indices included in the analysis. This estimated composite market index is the common risk factor which explains the co-movements of the stock indices and a portion of their returns.

2.1. The Dynamic Common Factor Model

To describe the dynamic common factor model, we will reference the framework and approach adopted by Stock and Watson (1991). In their paper, they wrote a macroeconomic model as a dynamic common factor model, estimated the parameters by maximum likelihood, and interpret the unobserved factor as a leading economic indicator. In their description of the model, they describe the "single index" model, its state-space representation, and the estimation of the likelihood function using the Kalman filter; we will also follow this format.

For our description of the dynamic common factor model, we will use the following notation. Also, refer to section 3 for a full description of the data. There are I stock indices, indexed by i individual stock indices. There are T time periods indexed by t, which represents the index's return at date t.

The variable $y_{i,t}$ is the logarithmic stock index *i* daily return. Additional discussion of the data and stationary tests will follow in the data section. Each variable $y_{i,t}$ at time *t* can be decomposed into two stochastic components: the idiosyncratic component, $u_{i,t}$, and the common component, c. The idiosyncratic component represents movements which only impact the individual stock index. The common component is the latent time-series variable which can also be viewed as a composite market index for all stock indices included in the analysis. Both the common and the idiosyncratic components will be modelled as having autoregressive processes of order q and p respectively. Percentage returns are the variables of interest and the model is represented as:

(1) $y_{i,t} = \beta_i + \gamma_i C_t + u_{i,t},$ (i = 1,2, ..., I)

(2) $c_t = \emptyset_1 c_{t-1} + \emptyset_2 c_{t-2} + \ \dots\ \emptyset_p c_{t-p} + \eta_t,$

(3) $u_{i,t} = d_{i,1} u_{t-1} + d_{i,2} u_{t-2} + \ \dots\ d_{i,q} u_{t-q} + \varepsilon_{i,t},$ (i = 1,2, ..., I)

The above equations are modelled with several identifying assumptions to ensure that the co-movement of the multiple time-series stock indices arises from the common factor c_t. First we assume that the innovations, $u_{i,t}$, are mutually uncorrelated with each other at all leads and lags for all (i = 1,2, ..., I). Secondly, as mentioned above, we also assume the covariance matrix of $u_{i,t}$ is diagonal.

$$Cov(u_{i,t}) = \mathrm{diag}(d_{i,1}(u_{t-1}), \dots, d_{i,q}(u_{t-q}))$$

and

$$E\begin{bmatrix}\eta_t \\ \varepsilon_{i,t}\end{bmatrix}[\eta_t\ \varepsilon_{i,t}{'}] \equiv \ \mathrm{diag}(\sigma_\eta^2, \sigma_{\varepsilon_1}^2, \dots \sigma_{\varepsilon_I}^2)$$

Additionally, we normalize the scale of c_t by setting $\sigma_\eta^2 = 1$.

2.2. State Space Representation

In order to estimate the unknown parameters and extract estimates of the unobserved variable, c_t, we need to first transform equations (1) – (3) into state space form so that the Kalman Filter can be used to evaluate the likelihood function.

The basic idea, as described in Lütkepohl (2005), is that there exists an observed time series $y_1, y_2, \dots, y_T$ which depends upon an unobserved state variable z_t which is driven by a stochastic process. The relation between y_t and z_t is described by the measurement equation:

(4) $y_t = H_t z_t + v_t$

where H_t is a matrix that also depends on the period of time, t, and v_t is the observation error. The state vector or state of nature is generated as:

(5) $z_t = B_{t-1} z_{t-1} + w_{t-1}$

This equation is called the transition equation because it describes the transition of the state of nature from period $t-1$ to period t. The system of equations (4) and (5) is one form of a state space model.

In our model, the measurement equation refers to the observed stock market indices or variables, $y_{i,t}$. This equation relates the individual stock indices to elements of unobserved state vector which is the common risk factor or composite market index and comprised of c_t, u_t, and associated lags.

We start by obtaining the state or transition equation by combining (2) and (3) into the general standard state space form. We represent the variables $y_{i,t}, u_{i,t}, \varepsilon_{i,t}, c_t$, and η_t as vectors Y_t, U_t, E_t, C_t and N_t :

(6) $Y_t = \beta + Z\alpha_t$
(7) $\alpha_t = T\alpha_{t-1} + R\varsigma_t$

where:

$\alpha_t = (C_t', U_t{}')$

$\varsigma_t = (N_t', E_t{}')$

The matrix T denotes the time-invariant transition matrix and R, and Z denote the selection matrix. We denote the diagonal variance-covariance matrix of ς_t as ψ. The above discussion was provided in general terms for an in-depth understanding of the methodology; refer to the results in section 4 for an example of the matrix and state-space notation as it relates to the stock indices.

2.3. Estimation

Once in the state space form, the Kalman Filter provides the means of updating the state equation as new observations become available. Predictions are made by extrapolating these components into the future. Therefore, the Kalman Filter consists of two sets of equations to estimate the model: the prediction equations and the updating equations. As described in Lütkepohl (2005), the Kalman Filter recursively estimates the states, α_t, given observations, y_t, of the stock index. Under normality assumptions, the estimator of the state produced by the filter is the conditional expectation $E(\alpha_t|y_1, \dots y_t)$. The Kalman Filter also provides the conditional covariance matrix $Cov(\alpha_t|y_1, \dots y_t)$ which may serve as a measure for estimation or prediction uncertainty. For $t > T$ the estimator $E(\alpha_t|y_1, \dots y_t)$is a forecast at origin T. The computation of the estimators $E(\alpha_t|y_1, \dots y_t), t = 1, \dots, T$ is called filtering which distinguishes it from forecasting.

Let $\alpha_{t|t-1}$denote the estimate of α_t based on information $(y_1 \dots y_{t-1})$, $P_{t|t-1} = E\left(\left(\alpha_{t|t-1} - \alpha_t\right)\left(\alpha_{t|t-1} - \alpha_t\right)'\right)$, and recall that the diagonal variance covariance matrix of ς_t is ψ, then the two prediction equations are:

(8) $\alpha_{t|t-1} = T\alpha_{t-1|t-1}$
(9) $P_{t|t-1} = TP_{t-1|t-1}T' + R\psi R'$

We define the forecast error of the measurement equation (8) as $v_t = Y_t - \beta + Z\alpha_{t|t-1}$ and its variance-covariance matrix as $F_t = E(v_t\ v_t') = ZP_{t|t-1}Z'$. The updating equations of the Kalman Filter are:

(10) $\alpha_{t|t} = \alpha_{t|t-1} + P_{t|t-1}Z'F_t^{-1}v_t$

(11) $P_{t|t} = P_{t|t-1} - P_{t|t-1}Z'F_t^{-1}ZP_{t|t-1}$

Given initial estimates of T, R, ψ, and Z and starting values for $\alpha_{0|0} = 0$ and $vec(P_{0|0}) = (I - T\otimes T)^{-1}vec(\psi)$, the Kalman Filter equations (10) – (13) allow for recursive calculations to predict the state vector $\alpha_{t|t-1}$ and covariance matrix $P_{t|t-1}$.

The Gaussian log likelihood is then computed as:

(12) $L = \frac{1}{2}\sum_{t=1}^{T} v_t{}'F_t^{-1}v_t - \frac{1}{2}\sum_{t=1}^{T} \ln(\det(F_t))$

The Gaussian maximum likelihood estimates of the parameters are fond by maximizing L over the parameter space.

3. Data

3.1. Stock Indices

The following table describes the individual stock indices included in the dynamic common factor analysis:

Table 1 : Stock Indices and Stock Exchanges

Stock Index		Country	Exhanges	Time Zone	Hours of Operations
SP500	Standard & Poor 500	USA	NYSE & NASDAQ	UTC -5:00	9:30 am to 4:00 pm EDT
NASDAQ	Nasdaq Composite	USA	NASDAQ	UTC -5:00	9:30 am to 4:00 pm EDT
RUT	Russell 2000 Index	USA	NYSE & NASDAQ	UTC -5:00	9:30 am to 4:00 pm EDT
TSX	S&P/TSX Composite Index	Canada	TSX	UTC -5:00	9:30 am to 4:00 pm EDT
BOVESPA	BOVESPA Index	Brazil	BM&FBOVESPA	UTC -3:00	10:00 am to 5:00 pm BRT
MXN	Indice de Precios y Cotizaciones (IPC)	Mexico	Mexican Stock Exchange	UTC -6:00	8:30 am to 3:00 pm CDT

The Standard & Poor 500 (SP500) Index was first published in 1957 and includes 500 leading companies in leading industries of the US economy. The SP500 focuses on the large capitalization US equities market segment and covers approximately 75% of US equities.

The NASDAQ Composite Index was launched in 1971 and is a broad based index. The index includes over 3,000 securities which is the most of any US market index. Also, the index on average, trades more shares than any other US index. The NASDAQ includes companies in all areas of business including technology, retail, communications, financial services, transportation, media and biotechnology. The index is calculated under a market capitalization weighted methodology.

The Russell 2000 Index measures the performance of the small-cap segment of the US equity universe. The index is a subset of the Russell 3000 Index representing approximately 8% of the total market capitalization of that index. It includes approximately 2,000 of the smallest securities based on a combination of their market capitalization and current index membership. The index is constructed to provide a comprehensive and unbiased small-cap barometer and is completely reconstituted annually to ensure larger stocks do not distort the performance and characteristics of the true small capitalization index.

The S&P/TSX Composite Index was established in 1977 and is an indicator of market activity for Canadian equity markets. The index is capitalization weighted, covers approximately 95% coverage of the Canadian equities market, and is the primary gauge of the Toronto Stock Exchange listed companies.

The Bovespa Index (Ibovespa) was established in 1968 and is the main indicator of the Brazilian stock market's average performance. The index is a gross total return index weighted by traded volume and is comprised of the most liquid stocks traded on the Sao Paulo Stock Exchange.

The Mexican IPC index (Indice de Precios y Cotizaciones) was established in 1978 and is a capitalization weighted index of the leading stocks traded on the Mexican Stock Exchange. The index was developed with a base level of .78 as of October 30, 1978.

3.2. Data Modification

The stock indices time-series data is the publicly available daily adjusted closing price, henceforth referred to as the closing price, from Yahoo Finance. The data was selected on the basis of public availability of the stock market indices and regional overlapping trading hours. For the dynamic common factor analysis the date range of April 28, 1993 to December 31, 2010 was used. The date April 28, 1993 is the first date where data was available for all six stock indices. It is necessary for the measurement of the data-series co-movement to ensure that all exchanges trade at the same time, to allow for common stock index movement. This is also confirmed by the stock exchange time zone and hours of operations in Table 1. Although, allowable trading periods do not line up exactly, for the majority of hours the operational hours overlap. This allows us to discern whether there exists a common factor to link all the indices returns.

Since the goal is to determine what proportion of index returns are from a common factor, a logarithm of the closing price was calculated. This allows us to determine the logarithmic daily return for each index by taking the first difference of each time-series. Also in section 3.4, we will show that first differencing is a necessary step to ensure the data is stationary.

Additionally, indices with missing closing prices, for a variety of reasons not investigated, were filled in with the previous day's closing price. This ensures that when the markets re-opened the return calculations are not comparing against zero for the previous day. For example, following the September 11, 2001 terrorist attacks the SP500 and NASDAQ indices did not trade for the period from the 11th to the 16th, reopening on the 17th. For the dates September 11 to 16, the closing price from September

10th was used; therefore, the return calculation for the 17th would correctly compare against the previous close on September 10th.

3.3. Normalization

For each stock index, the logarithmic daily return time-series was also normalized by subtracting the sample mean and the variance was standardized to one. This is a necessary step to ensure that each stock index receives equal weighting when the dynamic common factor analysis is conducted. If the data is not normalized, the estimation procedures will assign those stock indices with higher variances more weight. Since we do not have a reason to give one stock index a different weight the normalization ensures that each data series is treated equally.

3.4. Stationarity / Unit Root Testing

The dynamic factor model and likelihood theory used here requires the data to be stationary. A weakly stationary process has the property that the mean, variance, and autocorrelation (AC) structure do not change over time.

Data which is integrated has a unit root of 1 and is considered a non-stationary process. In order to determine whether each data set is stationary, they need to be individually tested. To accomplish this we will use the Augmented Dickey-Fuller (ADF).

The ADF test is applied as follows:

$$H_0 : \gamma = 0$$

$$H_1 : \gamma < 0$$

$$y_t = \mu + \beta t + \gamma y_{t-1} + \gamma_1 \Delta y_{t-1} + \cdots + \gamma_p y_{t-p} + \varepsilon_t$$

Failure to reject the unit root is taken as evidence that a unit root is present, that is, $\gamma = 0$

In order to use the ADF test, the lag length, p, needs to be determined. We applied the selection approach using the Schwarz's Bayesian information criterion (SBIC). In applying the ADF test, we first applied the test on the data series of the daily closing price time-series data for each stock index. The results indicated that all indices except for BOVESPA were non-stationary; therefore, first-differencing or using stock index return data is most appropriate. The results indicating that the logarithmic returns or first-difference stock index data is appropriate are as follows:

Table 2 : First Differenced Augmented-Dickey Fuller Test Results

Stock Index	SBIC	Optimal Lags	ADF Test Stat.
SP500	-6.34212	2	-102.381 ***
NASDAQ	-5.73677	1	-118.754 ***
RUT	-6.04563	2	-66.359 ***
TSX	-6.52216	1	-102.963 ***
BOVESPA	-4.91529	1	-79.082 ***
MXN	-5.75481	2	-59.24 ***

* significant at the 10% critical value
** significant at the 5% critical value
*** significant at the 1% critical value

As the ADF test statistic is rejected at a 1% significance level, a unit root is not present and the logarithmic returns for the indices are stationary.

3.5. Potential Sources of Bias

While the logarithmic stock market returns data are stationary, they may be biased for the following reasons. Table 3 summarizes the contemporaneous cross correlations for the daily logarithmic stock market return data. In general, the North American stock indices, SP500, NASDAQ, RUT and TSX, are highly correlated with correlations above 0.70. The table also shows large correlations within the USA, as the SP500 have large positive correlations with the two other American stock indices, the NASDAQ and RUT. The use of the four North American stock indices might cause a bias that the estimated unobserved common factor or composite market index tends to be dominated by the movements of the four stock indices.

Table 3 : Contemporaneous Correlations of the Stock Index Returns

	SP500	NASDAQ	RUT	TSX	BOVESPA	MXN
SP500	1.00					
NASDAQ	0.8655	1.00				
RUT	0.8634	0.8668	1.00			
TSX	0.7057	0.6604	0.6767	1.00		
BOVESPA	0.4719	0.4252	0.4337	0.4269	1.00	
MXN	0.5719	0.5369	0.5363	0.5066	0.5221	1.00

Another potential source of bias may be attributed to the serial correlation of disturbances in the observable variables. One explanation for serial correlation is that relevant factors are omitted from the regression model or the autoregressive factor structure is not correctly specified. In the strictest sense, if the model is correctly specified, the disturbance in the observable variables should be uncorrelated.

4. Results

4.1. Applied Methodology

In order to estimate the unobserved composite market index we first transform the single-index model for the six indices (SP500, NASDAQ, RUT, TSX, BOVESPA, MXN) into a state-space representation so that the Kalman Filtering technique can be applied.

In the estimation of the dynamic common factor we adopted a first order autoregressive specification for the common factor c_t, so that $p = 1$. For the idiosyncratic portion we also adopted a first order autoregressive structure for $u_{i,t}$, so that $q = 1$. At this point, we note that we also tested various other specifications for the common factor and idiosyncratic portion[1]. Based on the following results pertaining to the significance of the coefficients, the serial correlation of observable disturbances, and randomness of the innovations we decided that a simple model of AR(1) is most appropriate.

Single-Index Model:

$y_{i,t} = \gamma_i c_t + u_{i,t},$ (i = SP500, NASDAQ, RUT, TSX, BOVESPA, MXN)

$\mathrm{c}_t = \emptyset_1 \mathrm{c}_{t-1} + \eta_t,$

$u_{i,t} = d_{i,1} u_{t-1} + \varepsilon_{i,t},$

With the measurement and transition equations, the Kalman Filter is used to construct the likelihood function and to estimate the unobserved composite market index. The results of the dynamic common factor model are as follows:

Measurement Equations:

$y_{SP500,t} = 1.1536\mathrm{c}_t + u_{SP500,t}$

$y_{NASDAQ,t} = 1.2129 c_t + u_{NASDAQ,t}$

$y_{RUT,t} = 0.9951 c_t + u_{RUT,t}$

$y_{TSX,t} = 0.8357 c_t + u_{TSX,t}$

$y_{BOVESPA,t} = 0.4990 c_t + u_{BOVESPA,t}$

$y_{MXN,t} = 0.6089 c_t + u_{MXN,t}$

---

[1] We tested an AR(2) structure for both the common factor and idiosyncratic portion and noted that results were insignificant. Also we considered the inclusion of a currency index which comprised of the weighted average of the foreign exchange value of the U.S. dollar against a subset of the broad index currencies but did not observe significant results and therefore excluded the currency index.

State / Transition Equation:

$c_t = -0.0485c_{t-1} + \eta_t;\ \sigma_\eta = 1\ (normalized)$

$u_{SP500,t} = 0.0294u_{t-1} + \varepsilon_{SP500,t}$

$u_{NASDAQ,t} = 0.0036u_{t-1} + \varepsilon_{NASDAQ,t}$

$u_{RUT,t} = -0.0354u_{t-1} + \varepsilon_{RUT,t}$

$u_{TSX,t} = -0.0646u_{t-1} + \varepsilon_{TSX,t}$

$u_{BOVESPA,t} = 0.0261_{t-1} + \varepsilon_{BOVESPA,t}$

$u_{MXN,t} = 0.0323u_{t-1} + \varepsilon_{MXN,t}$

4.2. Serial Correlation of Observable Disturbances

As mentioned in section 3.6, the serial correlation of observable disturbances is a possible source of bias or indicator of a poor fitting model. In order to test model fit, a test to determine the existence of serial correlation in the idiosyncratic error, $u_{i,t}$, will be performed.

Let e_y denote the one-step ahead forecast errors from the observable variables in the single-index model ($y = SP500, NASDAQ, RUT, TSX, BOVESPA, MXN$). That is $e_y = y_t - y_{t|t-1}$, where, $y_{t|t-1}$is computed using the Kalman Filter applied to an AR(1) model described in the above methodology.

The test for serial correlation of observable disturbances is performed as an F-test of the null hypothesis of joint insignificance for the coefficients of the one-step ahead forecast errors:

$H_0 :\ \gamma = 0$

$H_1 :\ \gamma \neq 0$

$e_{y,t} = \ \gamma_1 e_{y,t-1} + \cdots + \gamma_p y_{t-p} + \varepsilon_t$

Table 4 are the p-values from the regression of e_y with a, $\mathrm{p} = 1$, autoregressive lag structure for the common factor and q= 1 for the idiosyncratic error.

Table 4 : Serial Correlations of Observable Disturbances for an AR(1) Idiosyncratic Error Structure

	Dependent Variable					
Regressor	eSP500	eNASDAQ	eRUT	eTSX	eBOVESPA	eMXN
eSP500	0.0073 ***	0.1860	0.6097	0.0000 ***	0.0469 **	0.0000 ***
eNASDAQ	0.2016	0.3880	0.3018	0.0000 ***	0.0441 **	0.0000 ***
eRUT	0.1435	0.6652	0.8883	0.0000 ***	0.027 **	0.0000 ***
eTSX	0.0000 ***	0.0083 ***	0.0001 ***	0.0005 ***	0.9496	0.1686
eBOVESPA	0.5768	0.3937	0.4173	0.0000 ***	0.2302	0.0342 **
eMXN	0.8771	0.3851	0.0570	0.0000 ***	0.0000 ***	0.0000 ***

* significant at the 10% critical value
** significant at the 5% critical value
*** significant at the 1% critical value

Based on the results in Table 4, the null hypothesis that the coefficients on the lagged forecast errors are zero, is rejected for approximately half the stock indices. For the variables e_{SP500}, e_{NASDAQ} and e_{RUT} the results are mostly satisfactory as for the majority of the regressors the null hypothesis of the joint statistical insignificance of the lags cannot be rejected. However, it can be seen that when e_{TSX}, $e_{BOVESPA}$ and e_{MXN} are used as dependent variables the null hypothesis is statistically significant.

The results for the indices TSX BOVESPA, and MXN suggest that the disturbances in the observed variables are forecastable by the lagged disturbances from itself and other indices. These results go against the assumption that all errors are not correlated in all leads and lags. In particular, the TSX stock index's forecasted errors show persistence in the lagged dependent variable from all indices. This suggests that the equation for TSX is not correctly specified. In an effort to determine if higher-order models would improve the fit of the model, we also estimated an autoregressive of order two, $q = 2$, model and subjected the results to the same null hypothesis F-test. The results exhibited the exact same results and therefore in the interest of a parsimonious model; we continued the analysis with an AR(1) idiosyncratic error structure.

To correct for the serial correlation, one alternative is to include lags of the estimated common variable c_t, in the equation for TSX or we could increase the lag specification for $u_{i,t}$. Since the table suggests satisfactory results for the majority of the serial correlation test, the two suggested remedies are computationally very expensive, the econometrics software used, STATA, is limited in its ability to model custom equations, and the dynamic common factor we are suggesting is only a framework to test the significance of a common market factor; we will only raise the misspecification as a possible issue for future research.

4.3. Innovations are White Noise

In order to investigate whether the, $q = 1$, lag structure is robust enough to account for the serial correlation in the idiosyncratic variable, $u_{i,t}$, an appropriate test is conducted. The correlations in the data are examined to determine whether they are significantly different from zero. The test for autocorrelation is used to detect non-randomness in the data. The goal is to test whether the assumptions that innovations, $\varepsilon_{i,t}$, are white noise and random. If the residuals are not random, this is evidence that the serial correlation inherent in the idiosyncratic disturbances, $u_{i,t}$, are not fully accounted for in an AR(1) structure. Randomness in the disturbances can be determined by general tests called Portmanteau-tests on the following hypothesis:

H_0: $errors\ are\ random\ (white\ noise)$

H_1: $errors\ are\ not$

The alternative hypothesis is quite general and does not distinguish between different specifications of AR(q). In other words, there are a number of alternatives that could give rise to a rejection of the null hypothesis. At this point it should be noted that we also tested a second order autoregressive lag structure, $q = 2$, and noted that the differences to $q = 1$ were not materially different. Therefore, in the interest of a parsimonious model, the results of the Portmanteau tests on the innovations for, $q = 1$ lag structure are as follows:

Table 5 : Portmanteau Test for White Noise

Innovations	Q Statistic	Prob > chi2(12)
SP500	41.4051	0.0000 ***
NASDAQ	14.8426	0.2502
RUT	19.1552	0.0849 *
TSX	61.3476	0.0000 ***
BOVESPA	29.6701	0.0031 ***
MXN	44.2433	0.0000 ***

* significant at the 10% critical value
** significant at the 5% critical value
*** significant at the 1% critical value

The results in Table 5 indicates that at a 5% significance level only the NASDAQ and RUT stock indices have random innovations. This type of results would suggest that further modelling beyond AR(2) of the idiosyncratic portion of the dynamic common factor model is necessary. One method to generate better results is to model the individual stock indices idiosyncratic portions individually; increasing the number of lags for innovations which are not white noise. However, due to modelling limitations of the STATA software, varying the lag structures of the idiosyncratic portions is not possible.

Secondly, increasing the lag structure for all stock indices is computationally very expensive and also not feasible for the purposes of this paper. We therefore proceed with the specified AR(1) for structure for the idiosyncratic variable, $u_{i,t}$, but raise this misspecification as an issue for future research.

4.4. Dynamic Common Factor Model

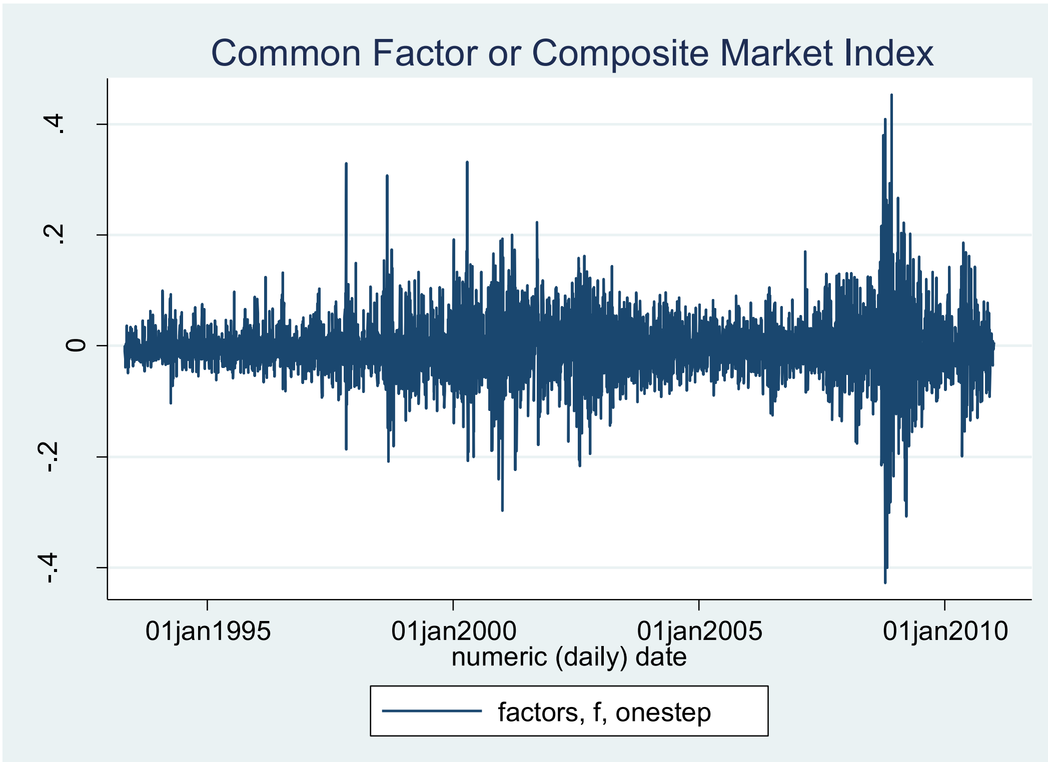

Figure 1: Common Factor for Logarithmic Daily Stock Index Returns

Figure 1 plots the estimated common factor or composite market index's daily returns. The composite index's returns are representative of the movements of the global equity markets. The returns show the volatility in the returns prior to the year 2000 and the large downward spike in 2001, which mirrors the technology-boom-bust recession. The composite market index also shows the effects of the most recent financial crisis with the returns spiking upwards and then downwards in 2008.

Recall that the composite market index was estimated using the Kalman Filter to estimate a model with an AR(1) structure for both the common and the idiosyncratic factor:

Table 6 : Parameter Estimates of Model

	$\emptyset\_1$	γ_i	$d\_(i,1)$
Common	0.0485		
	(0.0130)		
	0.0000 ***		
SP500		1.1536	0.0294
		(0.0116)	(0.0193)
		0.0000 ***	0.1290
NASDAQ		1.2130	0.0036
		(0.0125)	(0.0171)
		0.0000 ***	0.0410 **
RUT		0.9951	-0.0354
		(0.0102)	(0.0173)
		0.0000 ***	0.8320
TSX		0.8357	-0.0646
		(0.0121)	(0.0131)
		0.0000 ***	0.0000 ***
BOVESPA		0.4990	0.0261
		(0.0119)	(0.0126)
		0.0000 ***	0.0390 **
MXN		0.6089	0.0322
		(0.0116)	(0.0128)
		0.0000 ***	0.012 **

* significant at the 10% critical value
** significant at the 5% critical value
*** significant at the 1% critical value

As mentioned in the methodology section, in the estimation we normalize the scale of c_t by setting $\sigma_\eta^2 = 1$. Setting the variance of the innovation to unity affects the absolute magnitudes of the impact coefficients. Therefore, we will base our magnitude analysis on variance decompositions in section 4.5, rather than the magnitude of the estimated coefficients.

However, we can still review the statistical significance of the estimated parameters. The results of Table 6 shows that the coefficient, γ_i, which relates the individual indices to the unobservable common factor or the composite market index is significant for all indices. The impact of the common factor on the individual stock indices is statistically significant at the 1% level.

Additionally, we tested the null hypothesis that the coefficients are jointly zero, and the p-value of 0.0000 means that the individual stock indices coefficients are jointly significant. The existence of a statistically significant common factor, jointly and individually, for all individual stock indices would confirm the presence of co-movement among individual stock indices. In other words, the returns of each stock index can be attributed to a common factor and idiosyncratic factors.

The common factor also exhibits relatively low degree of autocorrelation as indicated by the statistical significance for the coefficient $\emptyset_{\mathbf{1}}$, with a value of 0.048. This suggests low persistence of the common stock returns on the individual stock indices. As mentioned in the introduction, with minimal persistence in the data the advantages of the common dynamic factor model the principal component analysis are reduced. This result suggests that the common component may be very close to the principal component leading eigenvector.

4.5. Correlation and Variance Decomposition

Based on the model estimated, we can measure the quantitative influence of variations in the common factors on fluctuations in the individual stock indices. By looking at the amount of volatility of each of the daily logarithmic stock index returns that are explained by the volatility of the common factor we can provide an economic interpretation to the magnitudes of the coefficient estimates in Table 6.

In keeping with the assumption that for each time-series the common factor and the idiosyncratic factor are orthogonal, the variance of each series can be decomposed into two terms:

(13) $\sigma_i^2 = \gamma_c^{i^2} \sigma_c^2 + \sigma_{i,u}^2$ where (i = SP500, NASDAQ, RUT, TSX, BOVESPA, MXN)

The parameter σ_i^2 represents the variance of an individual stock index logarithmic return. Recall from section 3.3 we previously normalized the variance of the logarithmic returns to unity, or $\sigma_i^2 = 1$. The impact coefficients, γ_i represents the estimated parameter, σ_c^2 represents the variance of the unobserved common factor, and $\sigma_{i,u}^2$ which represents the variance of each stock indices' idiosyncratic component or residuals.

As explained in the Gregory, Head and Raynauld (1997) paper, we can compute the estimates of R_c^i which measures the variance in the individual stock index logarithmic returns accounted for by the variation in the common factor. R_c^i is defined as the ratio of the variance of the common factor weighted by the appropriate impact coefficient, to the sum of the variances of the weighted common factor and the variance of the idiosyncratic component, $\sigma_{i,u}^2$, where (i = SP500, NASDAQ, RUT, TSX, BOVESPA, MXN).

Using these variances we can compute estimates of R_c^i as follows:

(14) $\widehat{R_c^i} = \dfrac{\frac{\hat{\gamma}_c^{i^2}}{1-\phi_c^2}}{\frac{\hat{\gamma}_c^{i2}}{1-\phi_c^2} + \frac{\hat{\sigma}_{i,\varepsilon}^2}{1-d_{i,1}^2}}$ where (i = SP500, NASDAQ, RUT, TSX, BOVESPA, MXN)

where $\sigma_{i,\varepsilon}^2$ is the estimated variance of the innovates to the idiosyncratic component of each individual stock index.

Table 7 : **Share of Variance Accounted for by Common Factors**

	$\widehat{R_c^i}$
SP500	47.33%
NASDAQ	46.52%
RUT	46.35%
TSX	36.06%
BOVESPA	19.96%
MXN	26.85%

The estimated share of variance accounted for by common factors, Table 7, provides quantitative meaning to the estimated impact coefficients given in Table 6. In the case of the US indices, the total logarithmic daily return variance accounted for by the common factor is similar and highest at approximately 50%. This suggests that no one single stock index is adequately able to capture the common component to all markets. For the other indices, approximately 37% of the variance of the logarithmic returns of the TSX stock index is explained by the common factor. The two southern stock indices, BOVESPA and MXN, the common factor's variance explains a lower amount of the index's logarithmic return variance. In addition to computing the amount of volatility in each stock index which is accounted for by the common factor volatility, we also compute the correlation between the common factor and the correlations between each individual stock index, as a measure of importance of the common factor.

Table 8 : Corrleation Matrix Between Idiosyncractic and Common Variables

	SP500	NASDAQ	RUT	TSX	BOVESPA	MXN	Common	Average
SP500	1.00							
NASDAQ	0.8655	1.00						
RUT	0.8634	0.8668	1.00					
TSX	0.7057	0.6604	0.6767	1.00				
BOVESPA	0.4719	0.4252	0.4337	0.4269	1.00			
MXN	0.5719	0.5369	0.5363	0.5066	0.5221	1.00		
Common	0.0719	0.0517	0.0455	-0.0693	-0.0092	-0.0496	1.00	0.0068

Table 8 presents the contemporaneous impact of the common factor on each individual stock index and the correlations between stock indices. The three US indices correlation with the common factor is 0.0719, 0.0517 and 0.0455 respectively. The TSX and MXN indices have a correlation of -0.0693 and -0.0496 respectively. In contrast, the BOVESPA index has a relatively low degree of correlation with the common factor at -0.0092. This is not surprising as the variance decomposition suggested that the common factor's variance for BOVESPA accounted for the lowest, among the indices, amount of index's variation. Overall, the contemporaneous correlation between the individual stock indices and common factor is low, with an average of 0.0068.

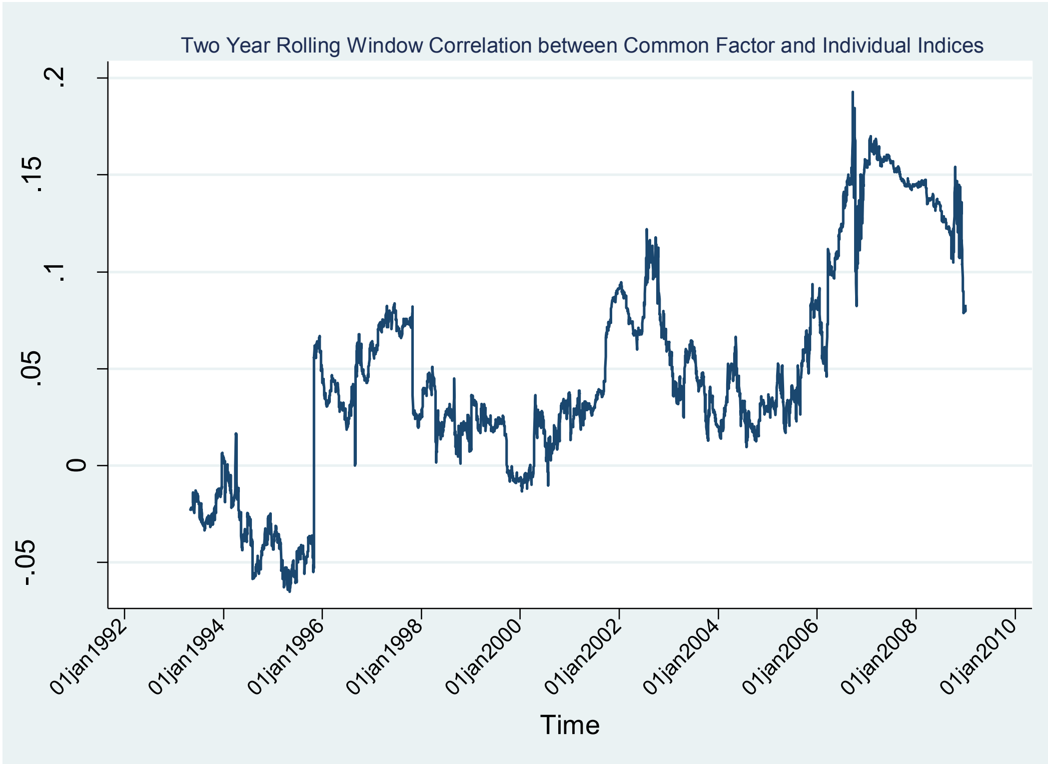

Figure 2: Average Two-Year Rolling Window Correlation between Common Factor and Individual Stock Indices

In Figure 2, we plot the average two year rolling correlation between the common factor and individual stock indices. The correlation was calculated as the average correlation between the common factor and the stock indices over a rolling window of two-year segments. As depicted graphically, on average, the correlation between has increased over the sample time period. In the early to mid-1990s, the average correlation between the individual indices and common factor was relatively low. Between the mid-1990s and mid-2000s, the correlation oscillated between zero correlation and 0.10. From mid-2000s, the correlations started to increase from 0.05 and peaking just below 0.20 and declining to approximately 0.10 for the two-year correlation window ending December 31, 2010. Also, the spike and decline in correlation during this time period corresponds to the recent financial recession. The increase in contemporaneous correlation suggest that over the sample period, the logarithmic return of individual stock have become relatively more synchronized with the common factor; corroborating our previous findings that the common factor is an increasingly significant factor in explaining stock index returns.

4.6. Standard Beta vs. Across-Index Composite Beta

Recall that the beta coefficient is defined as the covariance of the asset's return with the market return, in effect it measures the effect of the stock market returns on individual asset returns. To compare the impact of beta on standard market indices (SP500 and NASDAQ) against the composite market index (common factor), we computed betas for stocks across various industries.

In order to determine the impact of the standard index or composite index we utilized a one factor model regressing (OLS with robust standard errors) an individual stock return , R_i, against the market risk premium $R_m - R_f$. The risk-free rate used, R_f, is the US Long-Term Composite Rate; which is the unweighted average of bid yields on all outstanding fixed-coupon bonds neither due nor callable in less than 10 years. The regression followed the same period of April 28, 1993 to December 31, 2010 and the risk-free rate used was the December 31, 2010 rate of 3.98% converted to a daily rate.

In other words, we utilized a one factor model as follows:

(15) $R_i = \alpha + \beta_i(R_m - R_f)$ where (i = XOM, MSFT, JNJ, BAC, WY, CSCO)

Table 9 : Beta Calculations

Stock		Index		Standard Market Index Alpha-1	Standard Market Index Beta-1	Composite Market Index Alpha-2	Composite Market Index Beta-2	Beta-1/Beta-2	Alpha-1/Alpha-2
XOM	Exxon Mobil Corporation	SP500	Energy	0.0003	0.7854	0.0004	0.0293	26.8444	0.65
				(0.030)	(0.0000)	(0.009)	(0.0000)		
MSFT	Microsoft Corporation	NASDAQ	Technology	0.00035	0.93025	0.00062	0.03408	27.2948	0.57
				(0.028)	(0.0000)	(0.006)	(0.0000)		
JNJ	Johnson & Johnson	SP500	Health Care	0.0003	0.5783	0.0004	0.0180	32.1518	0.76
				(0.011)	(0.0000)	(0.004)	(0.0000)		
BAC	Bank of America Corporation	SP500	Financials	0.0002	1.5295	0.0004	0.0135	113.3969	0.45
				(0.412)	(0.0000)	(0.0165)	(0.0000)		
WY	Weyerhaeuser Company	SP500	Materials	0.0002	1.0363	0.0003	0.0024	430.5180	0.50
				(0.398)	(0.0000)	(0.177)	(0.0000)		
CSCO	Cisco Systems, Inc.	NASDAQ	Technology	0.0005	1.4557	0.0008	0.0338	43.0848	0.68
				(0.025)	(0.0000)	(0.009)	(0.0000)		

Alpha-1: standard alpha calculation per defined market index
Alpha-2: alpha calculation the common variable as the composite market index
Beta-1: standard beta calculation per defined market index
Beta-2: beta calculation the common variable as the composite market index

The results of Table 9 indicate that movements in the standard stock index return have a greater impact on the individual stock returns compared to movements in the composite stock index. This is not surprising as these individual stocks may be a large component of their standard index, but a much smaller portion of the composite market index. In fact, the results indicate that stocks which operate globally may over-state the risk of movement from the "market index" if it is defined as the standard index (SP500 or NASDAQ). The exclusion of movements from other global indices under-state the risks attributed to foreign indices. If the market is defined as the composite market return then the individual stocks are 25x to 430x overstating the impact of a market index.

The above table also shows the alpha generated by individual stocks. Alpha is defined as the excess return attributed to a security in excess of the returns attributed to the market index. For individual stocks the alpha when measured against standard market indices are all less than the alpha when using a composite market index. This means that when using a composite index, the idiosyncratic risks of the individual stocks are higher (or under-stated) and underpriced when using a standard market index. Measuring market risk using a common composite index implies that more idiosyncratic risks can be eliminated using portfolio diversification techniques.

4.7. An Alternate World Index Model

In order to test the importance of overlapping trading hours, we modified the model to include several European and Asian stock indices. Without repeating much of the analysis above, we determine that a $p = 1$ and $q = 1$ model was most appropriate. We should note that we tested an AR(2) specification and both models suffer from the same modelling issues when the robustness checks for observable variable disturbances and randomness of innovations encountered above. Therefore, based on the principal of a parsimonious model we adopted a first order autoregressive specification for the common factor c_t, so that $p = 1$. For the idiosyncratic portion we also adopted a first order autoregressive structure for $u_{i,t}$, so that $q = 1$.

The new data included spans the same April 28, 1993 to December 31, 2010 time period with the data modification techniques. The model was modified to include only the SP500 and TSX to represent the North American indices and the new European and Asian indices are:

Table 10 : Stock Indices and Stock Exchanges

Stock Index		Country	Exhanges	Time Zone	Hours of Operations
SP500	Standard & Poor 500	USA	NYSE & NASDAQ	UTC -5:00	9:30 am to 4:00 pm EDT
TSX	S&P/TSX Composite Index	Canada	TSX	UTC -5:00	9:30 am to 4:00 pm EDT
FTSE	FTSE 100 Index	UK	London Stock Exchange	UTC +0:00	8:00 am to 4:30 pm BST
DAX	Deutscher Aktien Index	GER	Frankfurt Stock Exchange	UTC +1:00	9:00 am to 5:30 pm CEST
HS	Hang Seng Index	HK	Hong Kong Stock Exchange	UTC +8:00	9:30 am to 4:00 pm HKT
N255	Nikkei Index	JPN	Tokyo Stock Exchange	UTC +9:00	9:30 am to 3:00 pm JST

From the above table, the trading hours for the European indices, FTSE and DAX, exactly overlap with the one-hour difference and time zone and staggered hours of operations. In addition, the European indices have a few hours of trading overlap with the North American indices. The Asian stock indices, HS and N255, overlap each other for the majority of operating hours. The FTSE's trading hours do not overlap with any Asian index. The DAX's trading hours has a one-hour overlap with HS and none with N255.

Using the Kalman Filter is to construct the likelihood function and to estimate the unobserved composite market index to represent these broad indices and note that the coefficients are significant for the common factor and the idiosyncratic factor.

Table 11 : **Parameter Estimates of Model**

	$\emptyset\_1$	γ_i	$d\_(i,1)$
Common	0.0692 (0.0180) 0.0000 ***		
SP500		0.9610 (0.0189) 0.0000 ***	-0.3197 (0.0166) 0.0000 ***
TSX		0.8330 (0.0185) 0.0000 ***	-0.1626 (0.1715) 0.0000 ***
FTSE		1.0118 (0.152) 0.0000 ***	-0.2463 (0.0228) 0.0000 ***
DAX		0.9836 (0.1487) 0.0000 ***	-0.1257 (0.0252) 0.0000 ***
HS		0.4266 (0.0146) 0.0000 ***	-0.1670 (0.0163) 0.0000 ***
N255		0.4266 (0.0178) 0.0000 ***	-0.1521 (0.0164) 0.0000 ***

* significant at the 10% critical value
** significant at the 5% critical value
*** significant at the 1% critical value

We also perform variance decomposition to quantify the magnitude of the indices and the results are as follows:

Table 12 : **Share of Variance Accounted for by Common Factors**

	$\widehat{R_c^i}$
SP500	37.04%
TSX	37.75%
FTSE	48.39%
DAX	49.06%
HS	16.72%
N255	14.30%

From Table 12, the variance accounted for by the common factors is similar and grouped regionally. The common factor's variance accounts for almost 50% of the European indices variance, 37% of the North American indices variance and 15% of the Asian stock indices variance. Also note that the common factor variance percentage of European and North American stock indices are closer in range. This is not entirely surprising as there are a few hours of operational overlap. Therefore, while co-movement exists between the indices even though hours do not overlap (Asian versus North American and Europe), the co-movement is stronger for indices that do overlap (North American and Europe).

5. Conclusion

With increasingly globalized financial markets and an increasing trend of investing in diversified portfolios of different countries through low-cost index funds and ETFs, the dynamic common factor model framework is an effective method to extract an unobserved common factor. This common factor explains a portion of the returns of assets and can be interpreted as a composite market index which has been custom constructed to account for the risks in an investor's portfolio. In this paper, we applied the common factor framework to explain the daily logarithmic returns of stock indices which have overlapping hours of operation. The results show that the impact coefficients related to the common factor are individually and jointly significant. This means that the common factor explains the movement of the stock indices; and the common factor represents the co-movements between the indices. To provide economic interpretation to the coefficients, we applied a variance decomposition. The variance decomposition shows that the common factor's variance explains approximately 50% of the US stock indices variance. Additionally a two-year rolling window correlation analysis between the stock indices and the common factor was performed. This analysis shows that the correlation has increased over the sample period, and shows that the common factor has become increasingly important in explaining returns.

Our comparison of standard market indices with the common factor index shows that using a one factor CAPM model the idiosyncratic risk is under-stated the effect of the standard index is over-stated. The effect of the common index beta is much lower than the standard index betas when various individual stocks are compared. This was not a surprising result as the individual stocks have a much less impact on the composite index than its standard index. This result implies that finance practitioners may be under-diversifying risks if they are using a standard market index.

In extending our analysis, we also utilized the dynamic factor framework to extend the analysis to global indices with the inclusion of European and Asian indices. The results shows that the portion of variance accounted for by the common factor's variance are grouped for indices of similar geography. In addition, the indices which overlap trading hours, North American and European, have a relatively close share of variance accounted for by the common factor variance. This suggests that co-movements of indices which overlap is higher when compared to the Asian indices which do not overlap in trading hours.

However, there are areas for further research. The analysis of the co-movement in the stock indices can be extended by testing whether excess co-movement exists. This type of analysis is related to existing literature on stock price and commodity price co-movements. One is a paper by Pindyck and Rotemberg (1990) who posed the question whether stock prices move together too much. The paper examined the co-movement of individual stock prices and determined whether the co-movement could be justified by economic fundamentals. They reasoned that if all stocks moved together for reasons unrelated to fundamentals, the market will move more than is justified by fundamentals. For every model and grouping of companies they determined that there exists an excess correlation of returns. Notably, they comment that their tests may be incomplete as they may have excluded some important macroeconomic variables from their model specification. Another paper by Shiller (1989) examined the co-movements in U.S. and U.K. stock prices and co-movements in dividends. They note the importance of their work by referring to October 19-20, 1987 when the level of stock prices in all major stock markets of the world made similar spectacular drops. Similar to Pindyck and Rotemberg, Shiller finds that there is significant co-movement between the U.S. and U.K. markets.

In addition, future research can be done to improve the dynamic common factor model. Our analysis of the serial correlation of the observed disturbances, which tests the autoregressive structure of the stock index returns, shows satisfactory results for half of the time-series data. Also, test of the lag structure of the idiosyncratic variable also indicated unsatisfactory results for several indices. This indicates that the model may not be perfectly specified and more robust models with increased lags or the inclusion of the lagged common variable can be considered in the future.

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