Secular Economic Changes in Bond Yields

Preliminary

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Abstract

We build a small-scale representation of the economy with secular and cyclical economic changes that are jointly determined by a small set of common structural shocks. Bond yields are influenced by the secular changes to the inflation and real rate endpoints π_t^* and r_t^*, but we impose that bond expected returns and term premium are stationary. We find that inflation and output shocks push the expectation and term premium components of long-term yields in opposite direction, which mutes the transmission from the short-rate response to long-term yields. However, short rate shocks push expectation and term premium in the same direction, which amplifies the transmission. The relative contribution of each shock changes over time. Hence, once we account for the effects of secular economic changes, we recover a complex cyclical relationship between the short-term rate and long-term yields.

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1 Introduction

Interest rates are best described using a mix of secular and cyclical changes. The evidence shows, notably in Kozicki and Tinsley (2001), that since the 1980s, inflation exhibited cyclical changes around a declining long-run level π_t^*. This long-run level has been stable around two percent since the end of the 1990s. The evidence also shows, notably in Laubach and Williams (2003), that since the 1990s, real rate and output growth exhibited cyclical changes around declining long-run levels r_t^* and g_t^*. Because the Federal Reserve responds in different ways to cyclical and secular changes, economists and bond investors need to disentangle the effects of structural shocks on the secular and cyclical components of yields.

We introduce a new framework based on three key features. First, we construct a small-scale representation of the economy to account for the endogenous relationships between the short rate, inflation and output, very much like benchmark structural VAR representation, but where each of the element of the VAR has cyclical and secular components. By contrast, benchmark structural VAR are based on stationary transformation of the data. One benefit from this representation is that econometric identification of the neutral real rate r_t^* is substantially improved by using the Fisher relationship $i^* = r^* + \pi^*$.

Second, we take the view that cyclical and secular changes are correlated because they share common structural shocks. The impact of structural shocks on the cyclical components are given by VAR dynamics with cross-equation restrictions but the impact on the secular components are permanent. In our baseline results, we use standard identification assumptions to recover structural shocks. We also implement cross-equation restrictions given in Laubach and Williams (2003), for comparability. However, our framework provides a laboratory to understand how different cross-equations restrictions and identification assumptions affect estimated secular and cyclical changes.

Third, we take the view that the variability of secular and cyclical changes is not constant. This is justified based on the consensus view that π_t^* was more volatile than r_t^* in the first half of our sample, but that r_t^* was more volatile in the second half. We let the data speak and we estimate how the variability of each component changes over time. In addition, we derive and impose restrictions that make sure that the variability of secular changes dies out with the forecast horizon. In standard unobserved component models, the variability of secular changes diverges as the forecast horizon increases. This severely over-estimates the uncertainty around the future, which affects standard analytical tools like the decomposition of variance.

Finally, we construct bond prices based on this small-scale model for the short rate, inflation and output. This construction relies on three key ingredients. First, the pricing equation relies on a general stochastic discount factor with linear prices of risk. Second, we impose economic restrictions guaranteeing that expected bond returns and the term premium are stationary (i.e, are not determined by secular changes). Finally, we introduce one additional shock to capture financial market variations. This shock is uncorrelated with macro-economic shocks.

We estimate the model using quarterly macroeconomic and bond market data for the US. The model does a good job fitting bond yields as well as long-horizon survey forecasts of macro variables. Overall, a few key results emerge. The long-run level of inflation declines in our sample and reaches a plateau near 2 percent. The volatility also declines. Early in the sample, the standard deviation of innovations to the long-run level of inflation is around 0.3 percent. However, this standard deviation is as low as a few basis points after 2000. Underlying this statistical description, we find a gradual disconnect of the long-run level of inflation from output shocks.

We also find a gradual decline in the long-run level of the real rate r_t^* starting in 2008, from a level above 2 percent, down to 1 percent in 2015 and close to 0.5 percent at the end of our sample. The volatility of innovation to the neutral real rate r_t^* is strongly counter-cyclical throughout our sample, reaching up to 0.25 percent during recessions and stabilizing around 0.1 percent in expansions. The structural decomposition shows that inflation and output shocks explain little of the recent decline, but that shock to the policy rate uncorrelated with inflation and output drives the decline in r_t^*. The recent decline is not associated with higher volatility but, instead, is driven by the extended period with a very low short rate.

Estimates of the model recover a stationary term premium driven by structural shocks. We find that macroeconomic shocks typically explain around half of the variability in yields at the quarterly horizon. However, this share rises above 90 percent at longer horizons. The results show a complex relationship between changes to the short rate and long-term yields. We find that inflation and output shocks lift the expectations component of yields but compress the term premium. By contrast, short rate shocks that are uncorrelated with output and inflation shocks also lift the term premium significantly. One interpretation of this results is that inflation and output shocks are considered good news for the economy, in which case the term premium mitigates the transmission to long-term yields, but that short rates shocks are considered bad news to the economy, in which case the term premium magnifies the transmission of shocks to long-term yields. The contributions of output, inflation and short rate shocks to the variance of yields vary over time. The share of the term premium variance attributed to short rate shocks is sometimes very low, less than 5 percent, but reaches beyond 60 percent at several occasions in our sample. Hence, once we account for the effects of secular economic changes, we recover a complex cyclical relationship between the short-term rate and long-term yields.

The cyclical components of the model exhibit large and significant impact of output gap on the inflation gap (the Phillips curve) as well as a large and significant impact of the real rate on the output gap (the IS curve). We argue that accounting for secular trends is the key features underlying this results. The magnitude of these coefficients is typically much smaller in similar models that only embeds cyclical features.

Section 2 introduces our small-scale model of the economy with secular and cyclical components and details our specification or bond prices. Section 3 provides details of the estimation methods. The likelihood of the macroeconomic data is available in closed form because we number of stochastic shocks is the same as the number of observable variables. Section 4 discusses the results.

2 Model

2.1 Conditional Mean

We develop a small-scale representation of the economy based on the Beveridge-Nelson decomposition of the short rate i_t, the inflation rate π_t and output y_t:

$$\begin{aligned} i_t &= i_t^* + \tilde{i}_t \\ \pi_t &= \pi_t^* + \tilde{\pi} \\ y_t &= y_t^* + \tilde{y}_t, \end{aligned} \tag{1}$$

or, for brevity, together in the vectors $M_t = M_t^* + \tilde{M}_t$.

The macro variables M_t are observable while the decomposition in terms of Beveridge-

Nelson components relies on the properties of their conditional mean dynamics. Define $\mathcal{E}_{t-1} = \mathcal{E}^*_{t-1} + \tilde{\mathcal{E}}_{t-1}$ the conditional mean of M_t, $\tilde{M}_t$ and M^*_t, respectively. For the cyclical (i.e., stationary) component $\tilde{M}_t$ we have

$$\tilde{\mathcal{E}}_{t-1} = \Phi \tilde{M}_{t-1}, \tag{2}$$

where $E\left[\tilde{M}_t\right] = 0$. The variables $\tilde{y}$, $\tilde{\pi}$ and $\tilde{i}$ are stationary and correspond to the output gap, the inflation gap and the short rate gap relative to the long-term shifting endpoints.[1] For the secular component M^*_t, the first two elements are integrated with order 1,

$$\begin{aligned} \mathcal{E}^*_{i,t-1} =& i^*_{t-1} \\ \mathcal{E}^*_{\pi,t-1} =& \pi^*_{t-1}, \end{aligned} \tag{3}$$

and, following Laubach and Williams (2003), the growth of potential output is integrated of order 1,

$$\begin{aligned} E_{t-1}\left[\Delta y^*_t\right] &= g^*_{t-1} \\ \mathcal{E}_{g,t-1} &= g^*_{t-1}. \end{aligned} \tag{4}$$

Specifically, y^* is potential output, while and i^*_t, π^*_t and g^*_t are the shifting endpoints for short rate, inflation and output growth, respectively, in the sense of Kozicki and Tinsley (2001). That is, for any variable x_t we have that

$$E_t[x_{t+h}] \underset{h}{\longrightarrow} x^*_t, \tag{5}$$

which correspond to forecasts at an horizon when cyclical influences dissipate.

2.2 Structural Shocks

The structural shocks are independent with identical distribution $\varepsilon_t \sim N(0, I)$. The identification of these shocks from the data is discussed below. The innovations

[1] The eigenvalues of the matrix Φ are strictly within the unit circle. Equation 2 embeds the family of stationary VAR(p) dynamics.

to M^* and $\tilde{M}$ are given by different linear combinations of the structural shocks:

$$M_t^* = \mathcal{E}_{t-1}^* + \Sigma_{t-1}^* \varepsilon_t \tag{6}$$
$$\tilde{M}_t = \tilde{\mathcal{E}}_{t-1} + \tilde{\Sigma}_{t-1} \varepsilon_t, \tag{7}$$

and therefore

$$M_t = \mathcal{E}_{t-1} + \Sigma_{t-1} \varepsilon_t, \tag{8}$$

with $\Sigma_t = \Sigma_t^* + \tilde{\Sigma}_t$. Finally, for the growth of potential output g^*:

$$g_t^* = \mathcal{E}_{g,t-1}^* + \sigma_{g^*}' \varepsilon_t. \tag{9}$$

2.3 Conditional Variance

The conditional variance are given by

$$\Sigma_t^* = \begin{bmatrix} v_{i,t}^* & 0 & 0 \\ 0 & v_{\pi,t}^* & 0 \\ 0 & 0 & v_{y,t}^* \end{bmatrix} \begin{bmatrix} \sigma_{i^*}' \\ \sigma_{\pi^*}' \\ \sigma_{y^*}' \end{bmatrix} = V_t^* \Sigma^* \tag{10}$$
$$\tilde{\Sigma}_t = \begin{bmatrix} \tilde{v}_{i,t} & 0 & 0 \\ 0 & \tilde{v}_{\pi,t} & 0 \\ 0 & 0 & \tilde{v}_{y,t} \end{bmatrix} \begin{bmatrix} \sigma_{\tilde{i}}' \\ \sigma_{\tilde{\pi}}' \\ \sigma_{\tilde{y}}' \end{bmatrix} = \tilde{V}_t \tilde{\Sigma},$$

where the matrix Σ^* and $\tilde{\Sigma}$ are unrestricted. The scalar $\tilde{v}_{\cdot,t} \in \{\tilde{v}_{i,t}, \tilde{v}_{\pi,t}, \tilde{v}_{y,t}\}$ and $v_{\cdot,t}^* \in \{v_{i,t}^*, v_{\pi,t}^*, v_{y,t}^*\}$ changes over time and allows the importance of structural shocks in cyclical and secular components to change over time.

The scaling factors follow $EGARCH(1,1)$ dynamics (see Nelson, 1991). To illustrate, in the case of the short rate $\tilde{i}_t$ we have

$$\ln\left(\tilde{v}_{i,t}^2\right) = \omega_i + \beta_i \ln\left(\tilde{v}_{i,t-1}^2\right) + g\left(z_{i,t}\right), \tag{11}$$

where

$$z_{i,t} \equiv \frac{\alpha_i' \varepsilon_t}{\sqrt{\alpha_i' \alpha_i}}, \quad g\left(z_{i,t}\right) = \sqrt{\alpha_i' \alpha_i} z_{i,t} + \kappa_i \left(|z_{i,t}| - E\left[|z_{i,t}|\right]\right),$$

with $\tilde{v}_{i,0} = 1$ and similarly for inflation gap and output gap, with additional parameters.

In the case of secular components, because of their unit root, we look for parameter restrictions in Equation 11 guaranteeing that the conditional variance of the secular components is well-behaved. To see this consider the case of i_t^* and suppose:

$$\sigma_{i,t}^* = v_{i,t}^* \sigma_{i^*},$$

where, as before, the $N \times 1$ vector σ_{i^*} is constant and the scalar v_t has EGARCH dynamics. This implies that

$$Var_t\left[i_{t+\tau}^*\right] = \sigma_{i^*}' \sigma_{i^*} \left(\sum_{j=1}^{\tau} E_t\left[v_{t+j-1}^2\right] \right),$$

where the coefficient is equal to 1 for each term in the sum because of the unit root in the process for i^*. Proposition 1 provides sufficient conditions for the convergence of $\lim_j Var_t\left[i_{t+j}^*\right]$.

Proposition 1 *The conditional variance converges,*

$$Var_t\left[i_{t+j}^*\right] \underset{j}{\longrightarrow} \frac{1}{\theta} v_t^2$$

, if $\beta = 1$ and $\omega < \bar{\omega}$, where $\theta \equiv 1 - e^{\omega - \bar{\omega}}$ and

$$\bar{\omega} \equiv \kappa \sqrt{\frac{2}{\pi}} - \ln \left(\begin{array}{c} \exp\left(\frac{\left(\sqrt{\alpha'\alpha} + \kappa\right)^2}{2} \right) \Phi_N\left(\kappa + \sqrt{\alpha'\alpha}\right) \\ + \exp\left(\frac{\left(\sqrt{\alpha'\alpha} - \kappa\right)^2}{2} \right) \Phi_N\left(\kappa - \sqrt{\alpha'\alpha}\right) \end{array} \right). \tag{12}$$

[*Appendix X*] provides the proof. A similar result is available from the authors in the context of asymmetric $GARCH(1,1)$ dynamics (Hentschel, 1995).

2.4 Pricing Nominal Bonds

We consider the general case where nominal bond prices depend on secular and cyclical components, but we impose restrictions such that the term premium is stationary for bonds with arbitrary maturity. Define $\overline{M}_t$:

$$\bar{M}_t' = [i^* \quad \pi^* \quad g^*]'.$$

We assume the following risk-neutral dynamic on macro-component

$$\begin{aligned} \bar{M}^*_{t+1} &= K^*_0 + \Phi^* \bar{M}^*_t + \Sigma^* \varepsilon^Q_{t+1} \\ \tilde{M}_{t+1} &= \tilde{K}_0 + \tilde{\Phi} \widetilde{M}_t + \phi_{Mf} f_t + \tilde{\Sigma} \varepsilon^Q_{t+1} \end{aligned} \tag{13}$$

where the scalar f_t follows stationary $AR(1)$ dynamics under the risk-neutral and the physical probability measures:

$$f_{t+1} = \phi_{Q,f} f_t + u^Q_{f,t+1} \tag{14}$$

$$f_{t+1} = \phi_f f_t + u^P_{f,t+1} \tag{15}$$

where $u^Q_{f,t+1} \sim^Q N\left(0, \sigma^2_{f,Q}\right)$ and $u^P_{f,t+1} = \tilde{\sigma}'_{fM}\left(\widetilde{M}_{t+1} - E_t\left[\widetilde{M}_{t+1}\right]\right) + \tilde{\sigma}_{ff}\varepsilon_{f,t+1}$ and where $\varepsilon_{f,t+1}$ is $i.i.d$ $N(0,1)$. Since it does not enter the short rate equation, f_t plays the role of a term premium factor and $\varepsilon_{f,t+1}$ is a risk premium shocks uncorrelated with structural macro shocks.

The yield to maturity of a zero-coupon bond is given by

$$\begin{aligned} Y^{(n)}_t &= -\frac{\ln\left(P^{(n)}_t\right)}{n} = -\frac{A_n}{n} - \frac{B^{*\prime}_n}{n}\overline{M}^*_t - \frac{\tilde{B}'_n}{n}\widetilde{M}_t - \frac{B_{f,n}}{n} f_t \\ &= a_n + b^{*\prime}_n \overline{M}^*_t + \tilde{b}'_n \widetilde{M}_t + b_{f,n} f_t. \end{aligned} \tag{16}$$

where, since the short rate can be written as

$$i_t = e'_1\left(\overline{M}^*_t + \widetilde{M}_t\right), \tag{17}$$

the initial conditions are $A_1 = 0$, $B^*_1 = \tilde{B}_1 = -e_1$ and $B_{f,1} = 0$. For maturities $n \geq 1$, the coefficients are given by

$$\begin{aligned}
A_{n+1} &= A_n + B_n^{*\prime} K_0^* + \tilde{B}_n' \tilde{K}_0 + \frac{1}{2}\begin{pmatrix} B_n^* \\ \tilde{B}_n \end{pmatrix}' \Omega_Q \begin{pmatrix} B_n^* \\ \tilde{B}_n \end{pmatrix} + \frac{\sigma_{f,Q}^2 B_{f,n}^2}{2} \\
B_{n+1}^{*\prime} &= B_n^{*\prime} \Phi_Q^* - e_1' = -e_1' \sum_{j=0}^{n} \left(\Phi_Q^*\right)^j \\
\tilde{B}_{n+1}' &= \tilde{B}_n' \tilde{\Phi}_Q - e_1' = -e_1' \sum_{j=0}^{n} \left(\tilde{\Phi}_Q\right)^j \\
B_{f,n+1} &= B_{f,n}\phi_{Q,f} + \tilde{B}_n' \Phi_{Q,Mf} = -e_1' \left[\sum_{j=0}^{n} \left(\frac{1-\phi_{Q,f}^{n-j}}{1-\phi_{Q,f}}\right)\left(\tilde{\Phi}_Q\right)^j\right] \Phi_{Q,Mf}.
\end{aligned}$$

In general, this specification means that all the secular components $\bar{M}_t$ determine bond yields. We impose the economic restriction that expected returns to holding bonds are stationary. Starting from the definition of expected returns to holding bonds in excess of the risk-free rate we get:

$$\begin{aligned}
& brp_{t\to t+n}^{(\tau)} \\
=\ & -(\tau-n)E_t\left[Y_{t+n}^{(\tau-n)}\right] + \tau Y_t^{(\tau)} - nY_t^{(n)} \\
=\ & A_{\tau-n} + A_n - A_\tau + B_{\tau-n}^{*\prime}\left(I - \left(\Phi_Q^*\right)^n\right)\overline{M}_t^* \\
& + \tilde{B}_{\tau-n}'\left\{E_t\left[\widetilde{M}_{t+n}\right] - \left(\tilde{\Phi}_Q\right)^n \widetilde{M}_t\right\} \\
& + B_{f,\tau-n}E_t\left[f_{t+n}\right] - \left(B_{f,\tau} - B_{f,n}\right) f_t. \qquad (18)
\end{aligned}$$

Proposition 2 follows directly from examining the loadings on $\bar{M}_t$:

Proposition 2 *The bond risk-premium is stationary (it only depends on the cyclical component) if and only if* $\Phi^* = I$.

Imposing $\Phi^* = I$ implies that the yield equation is now given by:

$$Y_t^{(n)} = i_t^* + a_n + \tilde{b}_n' \widetilde{M}_t + b_{f,n} f_t, \qquad (19)$$

the bond risk-premium per period is given by:

$$brp_{t\to t+n}^{(\tau)} = (1/n)\left\{\tilde{B}_{\tau-n}'\left(E_t\left[\widetilde{M}_{t+n}\right] - \left(\tilde{\Phi}_Q\right)^n \widetilde{M}_t\right) + B_{f,\tau-n}E_t\left[f_{t+n}\right] - \left(B_{f,\tau} - B_{f,n}\right) f_t\right\}, \qquad (20)$$

and the term premium is given by:

$$TP_t^{(n)} = a_n + \tilde{b}_n' \widetilde{M}_t + b_{f,n} f_t - e_1' E_t \left[(1/n) \sum_{k=0}^{n-1} \widetilde{M}_{t+k} \right]. \tag{21}$$

3 Estimation

3.1 Data

The sampling frequency is quarterly. Figure 1 shows the data. For the short rate i_t, we use the secondary market rate 3-Month Treasury Bill in percent. For the inflation rate π_t, we use compounded rate of change of the Personal Consumption Expenditures index excluding Food and Energy, annualized, in percent and seasonally adjusted. For output y_t, we use real gross domestic product, in log of billions and seasonally adjusted.[2] For yield data, we use the zero-coupon yields, in annualized percent from the GSW database Gurkaynak et al. (2006). We select yields with annual maturities between one and ten years. Figure 2 shows the 10-year yield that we use to estimate the data.

3.2 Identification

We impose several cross-equation restrictions in the cyclical dynamics for $\tilde{M}_t$. In the baseline results, we follow Laubach and Williams (2003) closely. First, we use the same reduced-form specification for inflation,

$$E_{t-1} \left[\tilde{\pi}_t \right] = b_1 \tilde{\pi}_{t-1} + b_2 \tilde{\pi}_{t-2,4} + b_y \tilde{y}_{t-1}, \tag{22}$$

which links expected inflation to lags of inflation and to the output gap. We also use the same reduced-form specification of the IS curve,

$$E_{t-1} \left[\tilde{y}_t \right] = a_1 \tilde{y}_{t-1} + a_2 \tilde{y}_{t-2} + \frac{a_r}{2} \sum_{j=1}^{2} \tilde{r}_{t-j}, \tag{23}$$

where $\tilde{r}_t$ is the cyclical component of the real rate. As in Laubach and Williams (2003), Equation 23 provides econometric identification of the real rate. However, even without this restriction, identification of the real rate follows directly from $\tilde{r}_t \equiv$

[2] Short rate, inflation and output data are available from the Federal Reserved of St-Louis website

$\tilde{i}_t - E_t[\tilde{\pi}_{t+1}]$.

The small-scale representation also includes the short rate i_t, for which we use a modified version of the Taylor rule,

$$\tilde{i}_t = \delta_\pi \tilde{\pi}_t + \delta_y \tilde{y}_t + \tilde{z}_t, \tag{24}$$

where the deviation $\tilde{z}_t$ from the classical rule is persistent: $E_{t-1}[\tilde{z}_t] = \phi_z \tilde{z}_{t-1}$. Together, Equations 22-24 implies that $\tilde{M}_t$ follows $VAR(5)$ dynamics given in the appendix.

There are three structural shocks ε_t that can be identified in this framework. The baseline model uses the common ordering identification assumptions, with output shocks first, inflation shocks second and short rate shocks last. We use a broad interpretation of short-rate shocks consistent with the econometric assumptions. Short rate shocks captures all that is left after the projections of the innovations on the output shocks and the inflation shocks.

3.3 Likelihood

We assume that one linear combination of yields is measured without error. This allows to invert latent financial market factor from the data, as in Chen and Scott (1993). Then the likelihood function of the data is available in closed-form, starting from initial values.

Let $(\tau_1, \tau_2, ..., \tau_J)$ be the set of bond maturities and $Y_t = (Y_t^{(\tau_1)}, ..., Y_t^{(\tau_J)})'$ be the yields to maturity for these bonds. We consider a portfolio of yields $\mathcal{P}_t \equiv WY_t$ that is priced exactly, where W is a $1 \times J$ row vector of weights.

$$\mathcal{P}_t = A_W + (W\iota_J)\, i_t^* + \tilde{B}_W' \widetilde{M}_t + B_{f,W} f_t,$$

which implies that we can recover f_t :

$$f_t = (1/B_{f,W}) \left[\mathcal{P}_t - \left(A_W + (W\iota_J)\, i_t^* + \tilde{B}_W' \widetilde{M}_t \right) \right],$$

where ι_J is a $J \times 1$ column vector of 1 and where

$$A_W = W \left[a_{\tau_1}, \cdots, a_{\tau_J}\right]$$
$$\tilde{B}_W = \left[\tilde{b}_{\tau_1}, \cdots, \tilde{b}_{\tau_J}\right] W'$$
$$B_{f,W} = \left[b_{f,\tau_1}, \cdots, b_{f,\tau_J}\right] W'.$$

[to be completed]

3.4 Survey Data

We use long-horizon survey forecasts to improve the identification secular and cyclical from each other. The Beveridge-Nelson decomposition relies on the difference in persistence between the secular and cyclical components. In practice, it is difficult to pin down in the data based on the one-step ahead likelihood. Intuitively, increasing the variance of secular changes may improve the fit but produces implausible estimates.

However, the implications for long-term forecasts are very different. At very long horizons, forecasts change essentially one-for-one with the secular components. We use this to identify variations in the secular and cyclical components from each other. We use recent results from Crump et al. (2016). They provide smoothed estimates of long-horizon forecasts for the short rate, inflation and GDP. The idea is that different sources offer survey forecasts at different frequencies and horizons, which can be pooled together to obtain smoothed estimates at a regular frequency and fixed horizons.

We add a measurement equations based on the survey forecast S_t^* available from Crump et al. (2016) which corresponds exactly to the very-long horizon forecast M_t^* in the model:

$$S_t^* = M_t^* + e_t, \tag{25}$$

where the measurement errors e_t are independent with normal distribution with diagonal covariance matrix Γ. The survey data that we use is shown in Figure 3. Crump et al. (2016) also provide estimate for the standard deviation of these measurement errors in their models. These reflect the sampling uncertainty that surrounds estimates S_t^* given all the observed survey data. Hence, we calibrate the estimate of standard deviation parameters in the matrix Γ to their results.

4 Results

4.1 Inflation Endpoint

Figure 5a presents the inflation endpoint π_t^* estimated with (i) a model with constant volatilities and (ii) a model with time-varying volatilities. The inflation endpoint starts around 6 percent at the beginning of our sample and quickly falls to a plateau around 4 percent for most of the 1980s. Starting in 1992, the endpoint of inflation gradually declines and reaches a lower plateau slightly above 2 percent around 1998. The models closely agree with each other but the case with time-varying volatilities provides a much smoother picture in the second half of the sample.

To understand why, Figure 5b shows the one-year ahead variance of the shifting endpoint $\text{var}_t(\pi_{t+1}^*)$. The results show how the inflation endpoint became anchored during this period. In the model with constant volatility, the standard deviation of annual changes to the endpoint π_t^* is 23 basis points, while the standard deviation of observed inflation is 52 basis points. The model with time-varying volatility draw a very different picture. Early in our sample, the annual volatility is slightly higher, around 30 bps, but it declines dramatically after 1990, reaching a very low level between 1 and 3 basis points after 2000.

One key benefit of our framework is that we can document how the contribution of different shocks evolves as the inflation endpoint becomes anchored. Figure 7a shows the one-year ahead variance decomposition of observed inflation over time. Early in the sample, when the inflation endpoint is elevated and volatile, the contribution of output shocks to the variance decomposition is around 40 percent. The contribution of inflation shocks is also around 40 percent and that of the short rate shocks is 20 percent. However, as the inflation endpoint becomes anchored and less volatile, we see that the contribution of output shocks falls gradually to reach essentially zero. By contrast, the contribution of inflation shocks expands and reaches 80 percent. Output shocks have become disconnected from the inflation endpoint.

4.2 Neutral Real Rate

Figure 6a presents the neutral rate estimate r_t^*. The endpoint averages 2 percent between the start of our sample and 2008. The endpoint then exhibits a gradual decline and reach a low level around 1.5 percent soon after the 1990-1991 recession and ten years after the start of our sample. This decline is also apparent in other estimate of the neutral rate (e.g., Laubach-Williams). From the lense of the model, this is due to the decline of the inflation endpoint despite lower level of interest rates.

The neutral endpoint then rises from 1993 until 2000, which corresponds to the acceleration of productivity growth diagnosed by Fed chairman Greenspan. The short rate was kept unchanged while output growth accelerated but inflation remained subdued. This acceleration was attributed to the long-awaited impact of new technology, and from the lenses of the model it is due to the lack of inflation response to output growth. The last long swing in the neutral rate rate starts in 2008 from 2 percent to 0.5 percent at the end of our sample. The exact final estimate depends on the model and ranges between 0.5 and 1 percent.

Figure 6b shows that the one-hear ahead volatility of the neutral rate is 14 basis points in the model with constant volatility. This hides substantial variability. Similar to the inflation endpoint, the volatility of the real rate endpoint is very low toward the end our sample, around 3 basis point for most of the decade following 2005.

Figure 7b shows the variance decomposition of the neutral rate over time. Early in the sample, output shocks explain the majority of r_t^* variations, but this share gradually decline over time. By contrast, short rate shocks explain less than 20 percent of r_t^* variations early in the sample, but explains 70 percent starting in 1997. This means that the changes to the neutral rate were driven by shocks to the short rate uncorrelated with shocks to output or to inflation. This does not mean that monetary policy shocks were driving the neutral rate. Instead, short rate shocks include all the other conditions to which the Federal Reserve responds when setting its policy.

4.3 Term Premium

Figure 7 shows the term premium for the 10-year zero coupon yield estimated in our model compared with the estimate from Adrian et al. (2013) available from the Federal Reserve Bank of New York. The estimate exhibits essentially the same cyclical variations, with the same peaks and throughs. The key difference is that the term premium is stationary in our framework. By contrast, the ACM estimate shows a downward trend from 4 percent at the beginning of our sample to less than 0 percent at the end of our sample. This trend in the estimate arises because the ACM model assumes that interest rates return to a constant averages. Therefore, the model interprets the long period with high interest rates early in our sample as a period with high term premium. Conversely, the model interprets the long period with low interest rates at the end of our sample as a period with low and negative term premium. This is not the case in our framework because these episodes are interpreted as periods where the endpoint i_t^* had changed.

Figure 9a shows the variance decomposition of the 10-year term premium in terms of financial market shocks and macro shocks (grouping together short rate shocks, inflation shocks and output shocks) and across horizons ranging from 1-quarter ahead to 5-year ahead (20 quarters). Macro shocks explain 50 percent of term premium variations at the quarterly horizon and more than 90 percent at the 2-year horizon and beyond. In other words macro shocks drive most of the cyclical variations that we observe in the term premium.

The importance of macro shocks to the term premium changes over time. Figure 9b shows the 1-year ahead variance decomposition over time. The contribution of financial market shocks is stable over time, typically less than 20 percent of the term premium variance. By contrast, the contribution of output, inflation and short rate shocks vary over time. The share of the term premium variance attributed to short rate shocks is sometimes very low, less than 5 percent, but reaches beyond 60 percent several times in our sample. From the time series of the term premium in Figure 7, it appears that the share of variance due short rate shocks peaks in period when the level of term premium rises.

We can look at the impulse response function of bond yields to gain some insights into the nature of short rate shocks, relative to output and inflation shocks. Figure 10a shows the responses of the 10-year expectation and term premium components of yields, separately, following inflation shocks. The response to output shocks is similar. The expectation component rises by 26 basis point on impact, unsurprisingly, and the effect propagates over several years. By contrast, the term premium component falls by 10 basis point on impact and then reverts to zero over several quarters. This result suggests that macro shocks are considered to be good news for bond investors. This is consistent with the pro-cyclical response by the Federal Reserve to economic conditions, which is made possible by low and anchored inflation. The term premium rises increases when the Fed responds to lower growth and inflation and decreases when the Fed increase the policy rate in response to the recovering conditions.

Figure 10b shows that the responses to short rate shocks are different. The response to the expectation component is much smaller than in the case of inflation shocks. However, the response of the term premium has the opposite sign than in the case of inflation shocks. Following a short rate shocks, the term premium increases by 5 basis point on impact and continue to rise over a few quarters, up to 15 basis point. This means that shocks to the short rate that are not correlated with output and inflation shocks are considered by news by investors. Take the case of a negative short rate shocks. The results that the term premium falls by as much as 15 basis point.

Therefore, the results show that changes to the short rate have a large impact on the term premium but with a sign that depends on the nature of the structural shocks. In addition. When macro shocks are relatively more important, the term premium tends to respond in the opposite direction as the short rate and mutes the transmission to long-term yields. By contrast, when short rate shocks are more important, the term premium tends to response in the same direction and to amplify the transmission to long-term yields.

5 Conclusion

We provide a framework where secular and cyclical changes to the short rate, inflation and output are jointly determined by a small set of structural shocks. This framework incorporates pricing equations for bond yields in terms of macro variables and one latent financial market factor. Estimates of the model recover a stationary term premium driven by structural shocks. We find that inflation and output shocks push the short rate and the term premium in opposite direction, which means that the transmission to long-term yield is muted. However, short rate shocks push the term premium in the same direction, which means that the transmission is amplified. Recovering the decline in the inflation and real rate endpoints is important to our results. Estimates of π_t^* and r_t^* are consistent with existing results. The inflation endpoint is well-anchored, it is mostly driving by inflation shocks and disconnected from output shocks. Future work is needed to understand why short rate shocks are different than inflation and output shocks.

Figure 1: Inflation and Output Growth

Inflation rate π_t: compounded rate of change of the Personal Consumption Expenditures index excluding Food and Energy, annualized, in percent and seasonally adjusted. Output growth Δy_t, log-change of seasonally adjusted real gross domestic product.

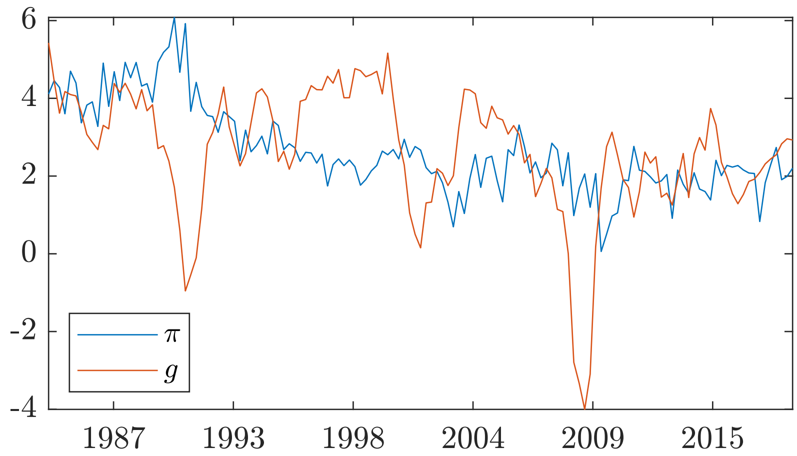


Figure 2: Nominal Yields

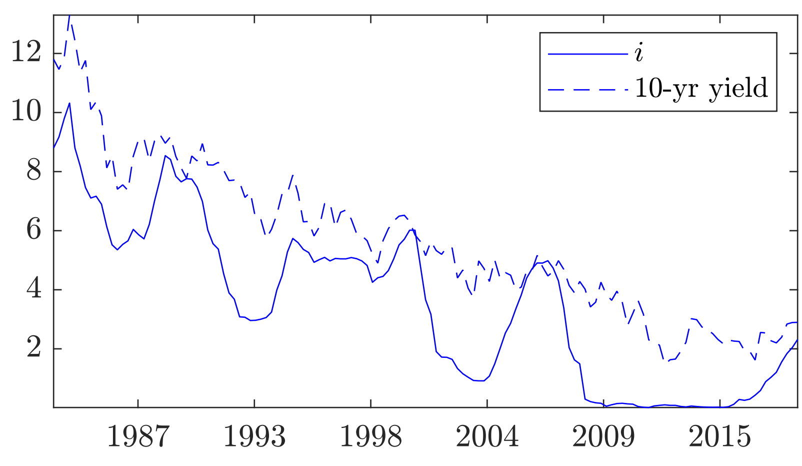


Figure 3: Long-Horizon Survey Forecasts

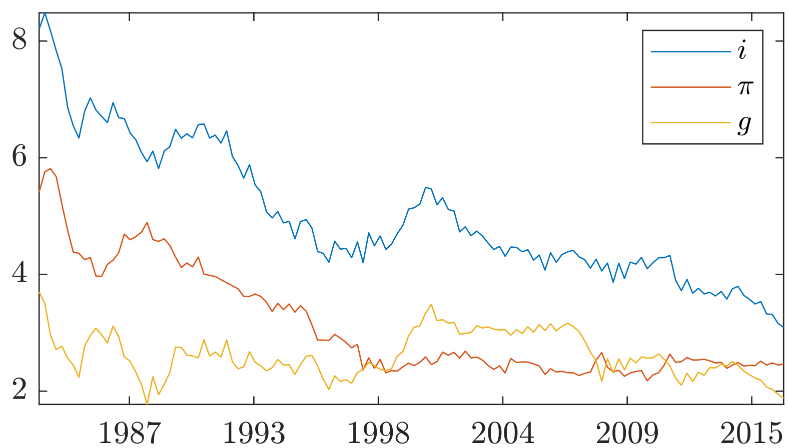
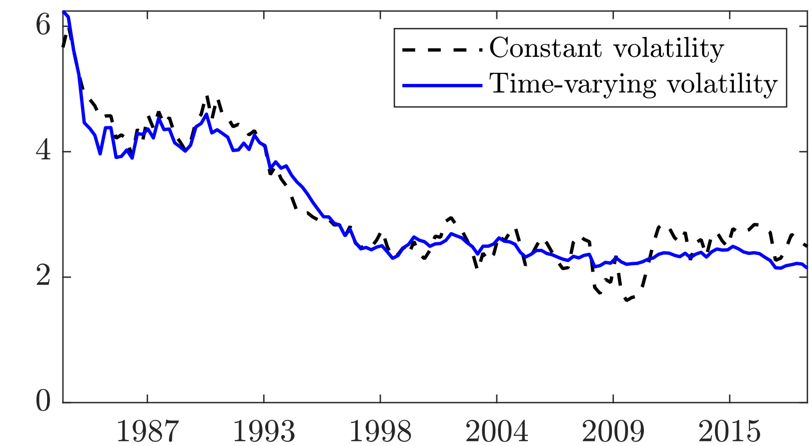


Figure 4: Inflation Endpoint

(a) Inflation shifting endpoint π_t^*



(b) Inflation shifting endpoint volatility

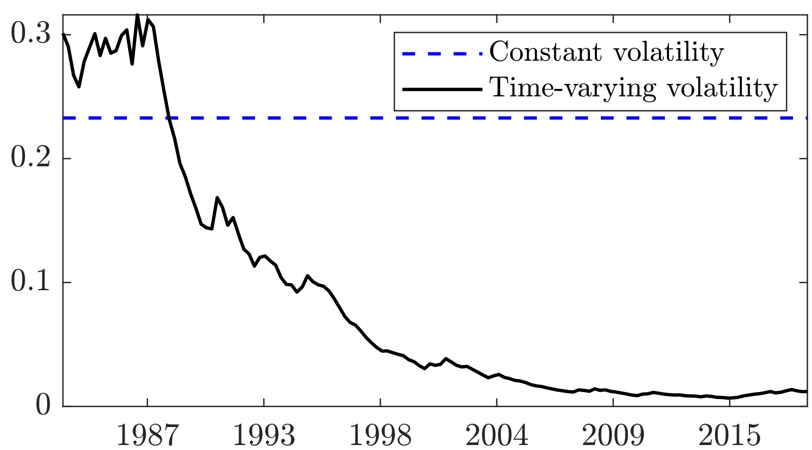
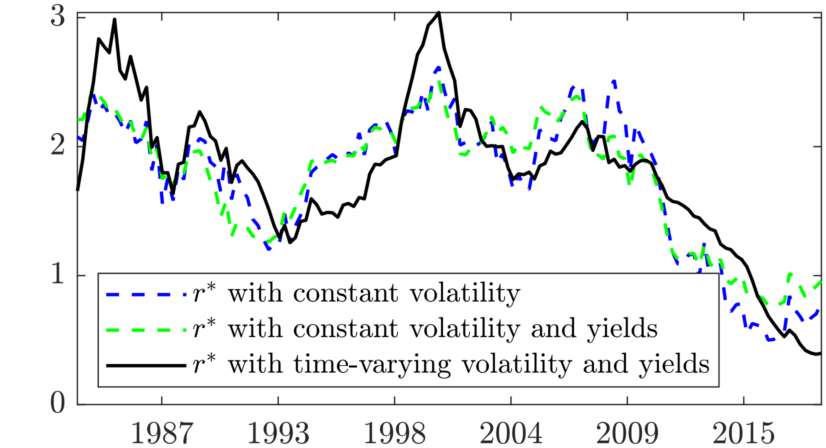


Figure 5: Real Rate Endpoint r^*

(a) Real rate shifting endpoint r^*_t



(b) Real rate shifting endpoint volatility

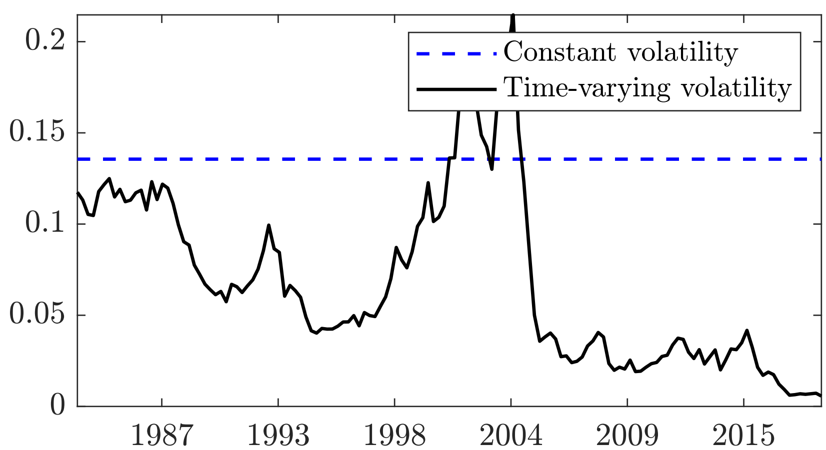
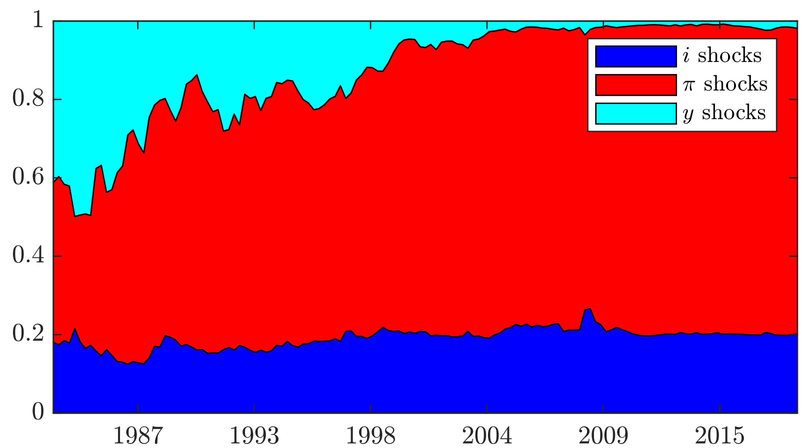


Figure 6: Variance Decomposition—π_t and r_t^*

(a) Inflation rate



(b) Neutral real rate

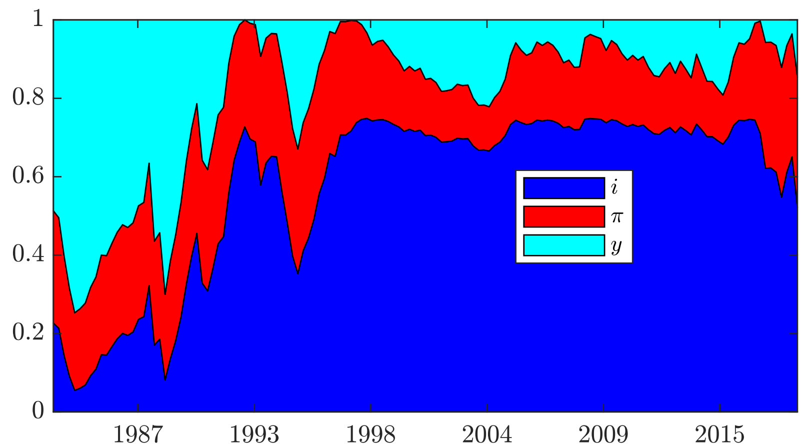


Figure 7: Term Premium

Term premium estimates compared with the estimates from Adrian et al. (2013), labeled ACM, available from the Federal Reserve Bank of New York.

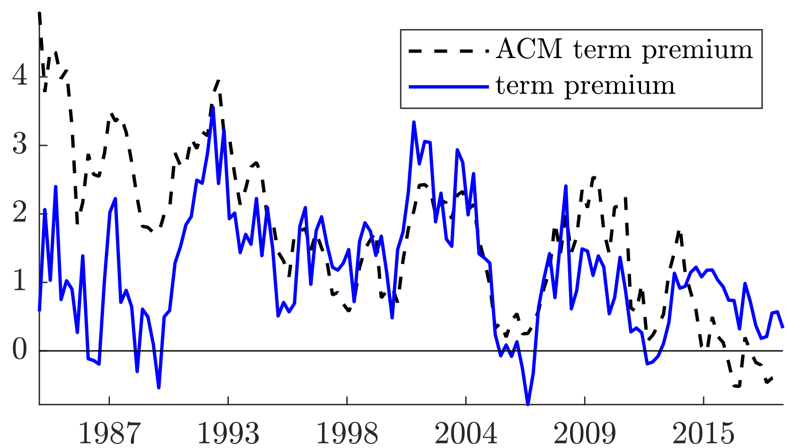
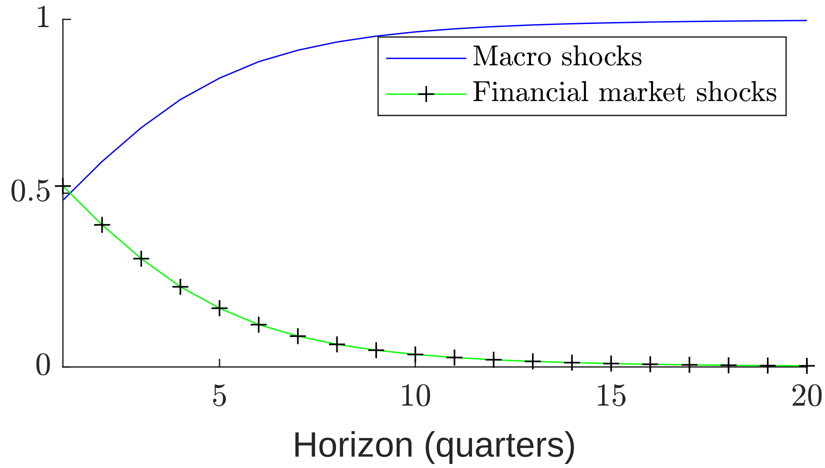


Figure 8: Variance Decomposition–Term Premium
Variance decomposition of the 10-year yield term premium. Panel (a) shows the decomposition in the first quarter of 2007 at horizons between 1 and 20 quarter. Panel (b) shows the decomposition over time at the 4-quarter horizon.

(a) Across horizons (January 2007)



(b) Over time (one-year horizon)

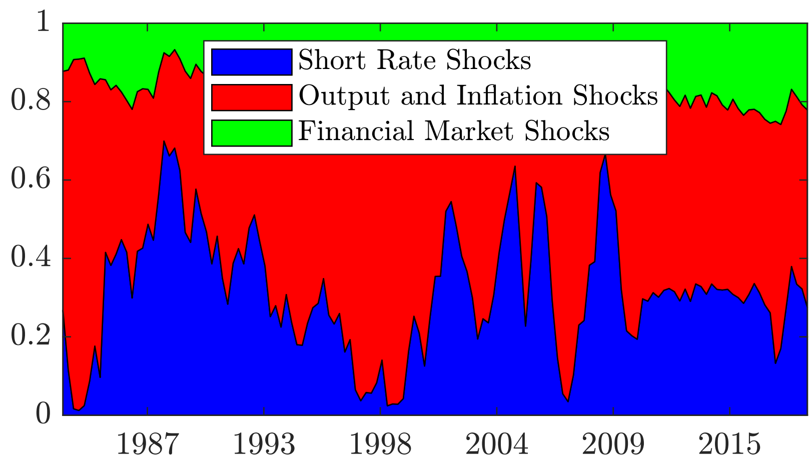
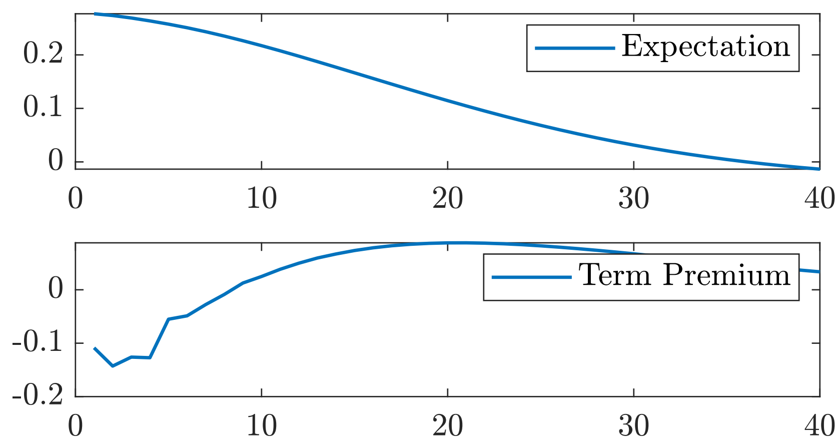
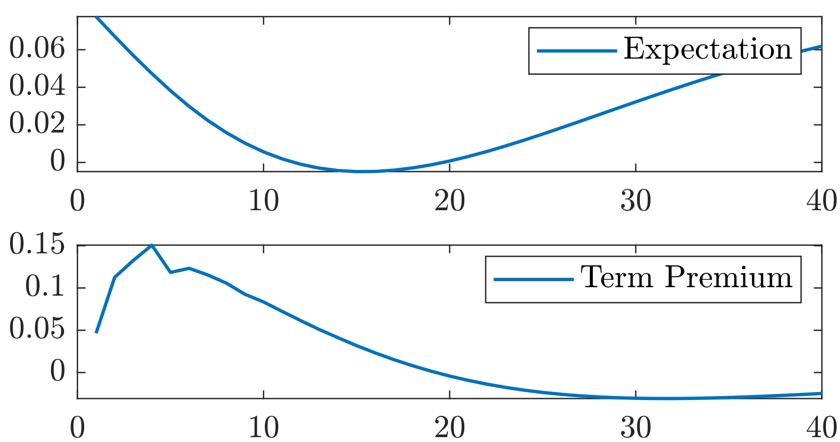


Figure 9: Impulse Response Function–Term Premium
Impulse response function of the expectation and term premium components of the 10-year yield, respsectively. Panel (b) shows the responses to inflation shocks (the response to output shocks is similar). Panel (a) shows the responses to short rate shocks.

(a) Inflation shocks



(b) Short rate shocks



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A Appendix

The cyclical components follow $VAR(5)$ dynamics with the following coefficients:

$$\Phi_1 = \begin{bmatrix} \left(\phi_z + \frac{a_r \delta_y}{2}\right) & \left(\delta_\pi b_1 - \frac{a_r b_1 \delta_y}{2} - \phi_z \delta_\pi\right) & \left(\delta_\pi b_y + \delta_y \left(a_1 - \frac{a_r b_y}{2}\right) - \phi_z \delta_y\right) \\ 0 & b_1 & b_y \\ \frac{a_r}{2} & -\frac{a_r b_1}{2} & \left(a_1 - \frac{a_r b_y}{2}\right) \end{bmatrix},$$

$$\Phi_2 = \begin{bmatrix} \frac{\delta_y a_r}{2} & \left(\frac{\delta_\pi b_2}{3} - \frac{\delta_y a_r}{2}\left(\frac{b_2}{3} + b_1\right)\right) & \delta_y \left(a_2 - \frac{a_r b_y}{2}\right) \\ 0 & \frac{b_2}{3} & 0 \\ \frac{a_r}{2} & -\frac{a_r}{2}\left(\frac{b_2}{3} + b_1\right) & \left(a_2 - \frac{a_r b_y}{2}\right) \end{bmatrix}, \quad \Phi_5 = \begin{bmatrix} 0 & -\frac{\delta_y a_r}{2}\frac{b_2}{3} & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{a_r}{2}\frac{b_2}{3} & 0 \end{bmatrix},$$

$$\Phi_3 = \begin{bmatrix} 0 & \left(\frac{\delta_\pi b_2}{3} - \delta_y a_r \frac{b_2}{3}\right) & 0 \\ 0 & \frac{b_2}{3} & 0 \\ 0 & -a_r \frac{b_2}{3} & 0 \end{bmatrix}, \quad \Phi_4 = \begin{bmatrix} 0 & \left(\frac{\delta_\pi b_2}{3} - \delta_y a_r \frac{b_2}{3}\right) & 0 \\ 0 & \frac{b_2}{3} & 0 \\ 0 & -a_r \frac{b_2}{3} & 0 \end{bmatrix}.$$